King Saud University
College of Science
Department of Mathematics

Mid-Term2/Summer/2016
Fulle Marks-25, Time 1h30m
6/11/1437

Q.1 (a) Find the larges interval on which the initial value problem

$$\frac{x+2}{x-2}y'' + \sqrt{9-x^2}y' + e^x y = \cos x, \quad y(0) = 1, y'(0) = 2$$

has a unique solution. [4]

(b) Determine whether the functions

$$f_1(x) = \cosh x,\ f_2(x) = \sinh x,\ f_3(x) = e^x$$

are linearly dependent or linearly independent on the interval $(-\infty, \infty)$. [4]

Q.2 (a) Find a homogeneous linear differential equation that has the general solution [4]

$$y = c_1 + c_2 x + c_3 e^{-x}\cos 2x + c_4 e^{-x}\sin 2x.$$

(b) If $y_1 = x$ is a solution of the differential equation

$$x^2 y'' - xy' + y = 0,$$

find its general solution. [4]

Q.3 Solve the differential equation [4]

$$y''' - y' = 2 - 6e^{-2x}.$$

Q.4 Solve the differential equation [5]

$$x^2 y'' + 5xy' + 3y = \ln x, \quad x > 0.$$