King Saud University: Mathematics Department Math-254
First Semester 1442 H Final Examination
Maximum Marks = 40 Time: 180 mins.

Name of the Student:―――――――――――――― I.D. No. ――――――――――

Name of the Teacher:―――――――――――――― Section No. ――――――――――

Note: Check the total number of pages are Six (6).

(20 Multiple choice questions and Two (2) Full questions)

The Answer Tables for Q.1 to Q.20 : Marks: 1.5 for each one ($1.5 \times 20 = 30$)

Ps. : Mark {a, b, c or d} for the correct answer in the box.

Q. No.	1	2	3	4	5	6	7	8	9	10
a,b,c,d										

Q. No.	11	12	13	14	15	16	17	18	19	20
a,b,c,d										

Quest. No.	Marks Obtained	Marks for Questions
Q. 1 to Q. 20		30
Q. 21		5
Q. 22		5
Total		40

Question 1: The number of iterations required to find the zero of $f(x) = x^3 - x^2 - 1$ in $[1, 2]$ accurate to within 10^{-6} using Bisection method is:

(a) 22 (b) 20 (c) 18 (d) None of These

Question 2: Using Newton's method the first approximation of the root of $x^3 - \alpha x^2 - 1 = 0$ is 2 when $x_0 = 1$, then the value of α is:

(a) 2 (b) 3 (c) 1 (d) None of These

Question 3: The equation $x^3 - 4x^2 - 3x + 18 = 0$ has a double root at $\alpha = 3$. Using quadratic convergent method with $x_0 = 0$, the first approximation of the root is:

(a) 12 (b) 6 (c) 3.5 (d) None of These

Question 4: Let $A = \begin{bmatrix} 1.001 & 1.5 \\ 2 & 3 \end{bmatrix}$, then the determinant of the lower-triangular matrix L of the LU factorization using Crout's method is:

(a) 1.001 (b) 0.300 (c) 0.003 (d) None of these

Question 5: The l_∞-norm of the Jacobi iteration matrix of the following linear system
$4x_1 - x_2 + x_3 = 7, \quad 4x_1 - 8x_2 + x_3 = -21, \quad -2x_1 + x_2 + 5x_3 = 15$; is:

(a) 0.5 (b) 0.625 (c) 0.6 (d) None of These

Question 6: The l_∞-norm of the first approximation of the solution vector obtained by Gauss-Seidel method starting with $\mathbf{x}^{(0)} = [0, 0, 0]^t$ for the following linear system
$4x_1 - x_2 + x_3 = 4, \quad 3x_1 + 8x_2 + x_3 = -21, \quad -2x_1 + x_2 + 5x_3 = 15$; is:

(a) 4 (b) 3 (c) 5 (d) None of These

Question 7: Let $A = \begin{bmatrix} 0 & \alpha \\ 1 & 1 \end{bmatrix}$ and $0 < \alpha < 2$. If the condition number $k(A)$ of the matrix A is 6, then α equals to:

(a) 0.2 (b) 0.8 (c) 0.5 (d) None of These

Question 8: Let $p_1(x)$ the linear Lagrange interpolation polynomial for the data $(2, 4)$ and $(5, \alpha)$. If the constant term in $p_1(x)$ is equal 6, then the value of α is:

(a) 2 (b) -1 (c) 1 (d) None of These

Question 9: If the quadratic Lagrange interpolation of a function f at $x = 1.4$ is given by $p_2(1.4) = Af(0) + Bf(1) + Cf(2)$, then the value of B is:

(a) -0.12 (b) 0.84 (c) 0.28 (d) None of These

Question 10: Let $x_0 = 2$, $x_1 = 2.5$, $x_2 = 4$ and $x_3 = 4.5$. If the best approximation of $f(x) = \frac{1}{x}$ at $x = 3$ using quadratic interpolation formula is $p_2(3) = 0.325$, then the value of the unknown point η in the error formula is equal to:

(a) 2.7859 (b) 2.9201 (c) 3.1472 (d) None of These

Question 11: If $x_0 = 0$, $x_1 = 1$, $x_2 = 3$ and for a function $f(x)$, the divided differences are $f[x_1] = 2$, $f[x_2] = 3$, $f[x_0, x_1] = 1$, $f[x_1, x_2] = \frac{1}{2}$, $f[x_0, x_1, x_2] = -\frac{1}{6}$. Then the approximation of $f(\frac{1}{2})$ using quadratic interpolation Newton formula is:

(a) 1.5417 (b) 4.1232 (c) 2.3481 (d) None of these

Question 12: Let $f(x) = x^2$ and $\alpha \neq 1$. The value of α such that $f[1, 1, \alpha] = \alpha^2$ is:

(a) 0 (b) 2 (c) -1 (d) None of These

Use the data in the following table to answer the Question 13 up to Question 16:

x	1.00	1.20	2.00	2.25	2.60	2.75	2.80	3.00
$f(x)$	0.4	1.5	2.03	2.7	3.7	4.5	5.2	α

Question 13: The best approximate value of $f'(2.5)$ using 3-point difference formula is:

(a) 2.6 (b) 3.6 (c) 1.6 (d) None of These

Question 14: If the best approximate value of $f'(3)$ using 3-point difference formula is 6, then the value of α is:

(a) 5.5 (b) 6.5 (c) 4.5 (d) None of These

Question 15: The best approximate value of $f''(2)$ using 3-point central difference formula is:

(a) 4.125 (b) 2.125 (c) 3.125 (d) None of These

Question 16: If $\alpha = 6$, then the best approximate value of $\int_1^3 f(x)\,dx$ using Simpson's rule is:

(a) 4.84 (b) 5.23 (c) 6.72 (d) None of These

Question 17: The number of subintervals required to approximate the integral $\int_1^3 (x^2 + \ln(4 - x))\,dx$ accurate to within 10^{-5} using the composite Simpson's rule is:

(a) 18 (b) 16 (c) 20 (d) None of These

Question 18: Given $y' + y = 2x$, $y(0) = -1$, the approximate value of $y(0.2)$ using Euler's method when $n = 2$ is:

(a) -1.01 (b) -0.9 (c) -0.79 (d) None of These

Question 19: Given $y' + xy = 0$, $y(0) = 1$, the approximate value of $y(0.2)$ using Taylor's method of order 2 when $n = 1$ is:

(a) 1.0 (b) 0.98 (c) 1.01 (d) None of These

Question 20: The actual error by using the Modified Euler's method of $y(0.1)$ where $y' = 2xy^2$, $y(0) = 1$, $n = 1$, and $y(x) = (1 - x^2)^{-1}$ is:

(a) 0.0001 (b) 0.0002 (c) 0.0003 (d) None of These

Question 21: Let $f(x) = \ln(x+2) + e^{-(x+2)}$ defined over the interval $[1, 2]$. Find the approximation of $\ln(3.8) + e^{-3.8}$ by using the equally spaced cubic Newton's polynomial on $[1, 2]$ if approximation by equally spaced quadratic Newton's polynomial on $[1, 2]$ is 1.3575 and fourth order divided difference is 0.0028. Compute the number of points to get the accuracy 10^{-6} using the equally spaced cubic Newton's polynomial on $[1, 2]$.

Question 22: Determine the number of subintervals n required to approximate the integral $\int_0^2 \frac{1}{x+4}dx$, with an error less than 10^{-4} using the composite Simpson's rule. Then approximate the integral $\int_0^2 \frac{1}{x+4}dx$.

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Question 21: Let $f(x) = \ln(x+2) + e^{-(x+2)}$ defined over the interval [1, 2]. Find the approximation of $\ln(3.8) + e^{-3.8}$ by using the cubic Newton's polynomial if approximation by quadratic Newton's polynomial is 1.3575 and fourth order divided difference is 0.0028. Compute the absolute error and the number of points to get the accuracy 10^{-6} using the cubic Newton's polynomial.

Solution. Since $h = (2-1)/3 = 1/3$; so $x_0 = 1, x_1 = 4/3, x_2 = 5/3, x_3 = 2$. Using quadratic polynomial $p_2(x)$ to find cubic Newton's polynomial $p_3(x)$, we have to use the formula

$$f(x) = p_3(x) = p_2(x) + f[x_0, x_1, x_2, x_3](x - x_0)(x - x_1)(x - x_2),$$

and this gives

$$f(x) = p_3(x) = p_2(x) + (0.0028)(1.8-1)(1.8-4/3)(1.8-5/3) \text{ and } f(1.8) \approx p_3(1.8) = 1.3575 + 0.00014 = 1.3576,$$

with possible absolute error

$$|f(1.8) - p_3(1.8)| = |1.3574 - 1.3576| = 0.0002.$$

Finally, as the upper bound of error in cubic polynomial is

$$|E_3| \le \frac{Mh^4}{24},$$

where

$$M = \max_{1 \le x \le 2} |f^{(4)}(x)| = \max_{1 \le x \le 2} \left| \frac{-6}{(x+2)^4} + e^{(-x-2)} \right| = 0.0243.$$

Since

$$\frac{Mh^4}{24} \le 10^{-6}, \quad h^4 \le \frac{(24) \times (10^{-6})}{M},$$

gives, $h \le 0.1773$ and $n = 5.6409 \approx 6$. Thus we need, 7 points, for the cubic interpolations.

Question 22: Determine the number of subintervals n required to approximate the integral $\int_0^2 \frac{1}{x+4}dx$, with an error less than 10^{-4} using the composite Simpson's rule. Then approximate the integral $\int_0^2 \frac{1}{x+4}dx$.

Solution. We have to use the error bound formula of Simpson's rule which is

$$|E_{S_n}(f)| \leq \frac{(b-a)}{180}h^4 M \leq 10^{-4}.$$

Given the integrand is $f(x) = \frac{1}{x+4}$, and we have $f^{(4)}(x) = \frac{24}{(x+4)^5}$. The maximum value of $|f^{(4)}(x)|$ on the interval $[0, 2]$ is $3/128$, and thus $M = \frac{3}{128}$. Using the above error formula, we get

$$\frac{3}{(90 \times 128)}h^4 \leq 10^{-4}, \quad \text{or} \quad h \leq \frac{2}{5}\sqrt[4]{15} = 0.7872.$$

Since $n = \frac{2}{h} = \frac{2}{0.7872} = 2.5407$, so the number of even subintervals n required is $n \geq 4$. Thus the approximation of the given integral using $h = \frac{2-0}{4} = \frac{1}{2} = 0.5$ is

$$\int_0^2 \frac{1}{x+4} \approx \frac{0.5}{3}\Big[f(0) + 4[f(0.5) + f(1.5)] + 2f(1) + f(2)\Big],$$

$$\int_0^2 \frac{1}{x+4} \approx \frac{1}{6}\Big[0.25 + 4(0.2222 + 0.1818) + 2(0.2) + 0.1667\Big] = 0.4055,$$

which is equal to the true value of the given integral $\alpha = \ln(1.5) = 0.4055$ up to 4 decimal places.

Solution of the Final Examination

The Answer Tables for Q.1 to Q.20 : Math

Ps. : Mark {a, b, c or d} for the correct answer in the box.

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a,b,c,d	b	c	a	c	b	a	c	c	b	a

Q. No.	11	12	13	14	15	16	17	18	19	20
a,b,c,d	a	c	b	b	a	a	c	c	b	a

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a,b,c,d	c	a	b	b	a	c	b	a	c	b

Q. No.	11	12	13	14	15	16	17	18	19	20
a,b,c,d	b	a	c	c	b	b	a	a	c	b

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Q. No.	11	12	13	14	15	16	17	18	19	20
a,b,c,d	c	b	a	a	c	c	b	b	a	c