King Saud University: Math. Dept. M-254 Final Exam.
Summer Semester (1433-1434 H) Time: 180 min. Max. Marks: 40

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**Question 1:** (5 + 4 + 5 + 4 + 4)

(a) Show that Newton's formula for the approximation of the cube root of 3 is

$$x_{n+1} = \frac{1}{3}\left(2x_n + \frac{3}{x_n^2}\right), \quad n \geq 0.$$

Find the approximation of the cube root of 3 using $x_0 = 1.5$ correct to 3 decimal places.

(b) Find the order of convergence of the iteration $x_{n+1} = \dfrac{x_n^2 + b}{2x_n}$, $n \geq 0$ as it converges to $\sqrt{b}$, $b > 0$.

(c) The equation $x^3 - 5x^2 + 4x - 3 = 0$ has one real root near $x = 4$ which is to be computed by the iteration (for $k \neq 0$)

$$x_{n+1} = \frac{3 + (k-4)x_n + 5x_n^2 - x_n^3}{k}, \quad k \text{ integer}, \quad x_0 = 4, \quad n \geq 0.$$

Determine which value of $k$ will give the faster convergence ? Using this value of $k$, find first approximation.

(d) Rearrange the following linear system

$$\begin{array}{rcrcrcr} x_1 & + & x_2 & - & 4x_3 & = & 4 \\ -5x_1 & + & 2x_2 & + & x_3 & = & -3 \\ x_1 & - & 10x_2 & + & x_3 & = & 27 \end{array}$$

such that the convergence of the Jacobi method is guaranteed. Use the Jacobi method to find the first two iterations, using the initial approximation $x^{(0)} = [-0.5, -2.5, -1.5]^T$. Compute an error bound for the approximation.

(e) Find the linear Lagrange polynomial passes through the points $(0, f(0))$ and $(\pi/2, f(\pi/2))$ to approximate the function $f(x) = 3\sin x$. Also, find a bound for the error in the linear interpolation of $f(x)$.

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**Question 2:** (4 + 5 + 5 + 4)

(a) If $f(x) = \ln(x+1)$, then find the value of the divided difference $f[1, 0, 0, 1]$.

Given the table of values

| $x$ | 0.6 | 0.9 | 1.2 | 1.4 | 1.5 | 1.8 | 2.2 | 2.4 | 3.0 |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 1.36 | 1.81 | 2.44 | 2.96 | 3.25 | 4.24 | 5.84 | 6.76 | 10.0 |

(b) Use the table to find the approximate value of the $f'(1.8)$ by the best three point formula.

(c) Use the table to compute the approximation of the integral $\int_{0.6}^{3} (x^2 + 1) f(x)\, dx$ using the best integration rule.

(d) Use Taylor's method of **order two** to approximate $y(1.8)$ for the initial value problem

$$\frac{y'}{x} - y = 0, \quad y(0.6) = 1.36, \quad n = 2.$$