King Saud University: Mathematics Department MAth-254
First Semester 1433-34 H Final Examination
Maximum Marks = 40 Time: 180 mins.

Name of the Student:———————— I.D. No. ————————

Name of the Teacher:———————— Section No. ————————

Note: Check please the total number of pages are Six (6).
(20 Multiple choice questions and Two (2) Full questions)

The Answer Tables for Q.1 to Q.20 : Marks: 1.5 for each one ($1.5 \times 20 = 30$)

Ps. : Mark {a, b, c or d} for the correct answer in the box.

Q. No.	1	2	3	4	5	6	7	8	9	10
a,b,c,d										

Q. No.	11	12	13	14	15	16	17	18	19	20
a,b,c,d										

Quest. No.	Marks
Q. 1 to Q. 20	
Q. 21	
Q. 22	
Total	

Question 1: Assume that the equation $x^3 - 3x^2 + 1 = 0$ has a root α in the interval $[k, 2k]$ where $k > 0$. If x_3 is the third approximation of this root by the bisection method for which $|\alpha - x_3| < 0.5$, then the value of k is:

(a) 1 (b) 3 (c) 2 (d) 4

Question 2: The fixed point iterative formula $x_{n+1} = g(x_n),\ n \geq 0$, will converge to $\alpha = 1$ provided that:

(a) $g(x) = 1 + \frac{1}{2}\ln(2 - x)$ (b) $g(x) = e^{x-1}$ (c) $g(x) = \sqrt{5x - 1} - 1$ (d) $g(x) = \dfrac{3x - 1}{x + 1}$

Question 3: Newton's iterative formula for approximating $\sqrt[3]{2}$ is:

(a) $x_{n+1} = 2x_n[1 + \frac{1}{x_n^2}]$ (b) $x_{n+1} = \frac{1}{3}[x_n - \frac{1}{x_n^2}]$ (c) $x_{n+1} = \frac{2x_n}{3}[1 + \frac{1}{x_n^3}]$ (d) $x_{n+1} = x_n[1 + \frac{1}{x_n^3}]$

Question 4: Suppose that $x_0 = 0$ and $x_1 = 1$ are approximate solutions for the equation $3x^3 - x - 1 = 0$. Then the next approximation of the solution using the secant method is:

(a) 1.5 (b) 0.5 (c) 1.0 (d) 2.0

Question 5: The value of k for which the scheme $x_{n+1} = 3 - (2 + k)x_n + kx_n^4$ will converge at least quadratically to $\alpha = 1$ is:

(a) $\dfrac{2}{3}$ (b) $-\dfrac{1}{3}$ (c) $\dfrac{1}{3}$ (d) $\dfrac{3}{2}$

Question 6: The first approximation for the root $\alpha = 0$ for the equation $e^{2x} - 2e^x + 1 = 0$ with $x_0 = 0.5$ by a quadratic convergent method is:

(a) 0.6065 (b) 0.1065 (c) 0.3037 (d) 1.065

Question 7: Let $J(x, y)$ be the Jacobian matrix of the nonlinear system $xy = 1, \quad x^3 + y^2 = 2$. Then at the point $(1, -1)$ the value of $\|J^{-1}\|_\infty$ is:

(a) 6.0 (b) 5.0 (c) 4.0 (d) 3.0

Question 8: Let $\hat{\mathbf{x}} = [0.1, 0.6]^T$ be an approximate solution for the linear system
$3x + y = 1, \qquad x + 3y = 2.$
If $\mathbf{r}$ is the residual vector with respect to $\hat{\mathbf{x}}$, then the $\|\mathbf{r}\|_\infty$ is:

(a) 0.1 (b) 0.2 (c) 1.2 (d) 0.9

Question 9: Let $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$, then the condition number of A is:

(a) 10 (b) 16 (c) 36 (d) 24

Question 10: Let T_J be the Jacobi iterative matrix for the linear system
$2x - y = 1, \qquad 2y - z = 1, \qquad x + y + 4z = 1.$
Then $\|T_J\|_\infty$ is:

(a) 0.75 (b) 0.25 (c) 0.5 (d) 1.0

Question 11: Let P_2 be the quadratic polynomial which interpolates a function f at the points 1, 2, and 3. If $P_2(2.5) = 4$ and $f[1,2,3,4] = 8$, then the approximate value of $f(2.5)$ using a cubic Newton interpolating polynomial is:

(a) -1.0 (b) 1.0 (c) -2.0 (d) 0.5

Question 12: Let $f(x) = \sqrt{x}$ and $\alpha > 0$. If $f[0, 1, \alpha^2] = -0.5$, then the value of α is:

(a) 1.0 (b) 12.0 (c) 7.0 (d) 32.0

Question 13: Let $P_2(x) = \sum_{i=0}^{2} f(x_i)L_i(x)$ be the quadratic Lagrange interpolating polynomial which interpolates a function f at x_0, x_1, x_2. If $L_0(x) = \frac{1}{8}(x^2 - 4x + 3)$ and $L_1(x) = \frac{1}{4}(-x^2 + 2x + 3)$, then $L_2(x)$ is:

(a) $-\frac{1}{8}(x^2 - 1)$ (b) $\frac{1}{8}(x^2 - 1)$ (c) $\frac{1}{8}(x^2 - 4x)$ (d) $-\frac{1}{8}(x^2 - 4x)$

Question 14: Let $M = \max_{x_0 \le x \le x_1} |f''(x)|$ for some function f. Then the error E in approximating $f(x)$ by a linear lagrange polynomial at x_0 and x_1 satisfies:

(a) $|E| \le \frac{hM^2}{8}$ (b) $|E| \le \frac{hM}{8}$ (c) $|E| \le \frac{h^2M}{8}$ (d) $|E| \le \frac{h^2M}{4}$

Question 15: If the piecewise function $f(x) = \begin{cases} 3x - 1, & 0 \le x \le \alpha \\ x + 2, & \alpha \le x \le 1 \end{cases}$ represents a linear spline, then the value of α is:

(a) $\frac{3}{2}$ (b) 2.0 (c) 3.0 (d) $\frac{2}{3}$

Question 16: Let $f(x) = e^x$. Then the **worst** approximate value of $f'(0)$ with $h = 0.2$ using 3-point numerical differentiation formula is:

(a) 0.984 (b) 0.101 (c) 1.107 (d) 1.0

Question 17: Let $h = 1$ and f is a function such that $f(x + h) = -f(x - h)$. If $f''(1) = 1.2$, then $f(1)$ is approximately equals to:

(a) -1.2 (b) -2.45 (c) -1.8 (d) -0.6

Question 18: The smallest number of subintervals needed to approximate $\int_0^1 \frac{1}{3-x}\,dx$ using composite Trapezoidal rule accurate within 10^{-3} is:

(a) 6 (b) 8 (c) 7 (d) 5

Question 19: For the initial-value problem $y' = 1 + \frac{y}{x}$, $y(1) = 1$, the approximate value of $y(1.2)$ using Euler's method with $h = 0.1$ is:

(a) 1.0 (b) 1.4 (c) 0.8 (d) 1.8

Question 20: For the initial-value problem $y' = x(y + 1)$, $y(0) = 0$, the approximate value of $y(0.2)$ using Taylor's method of order 2 with $n = 1$ is:

(a) 0.02 (b) 0.2 (c) 2.0 (d) 0.002

King Saud University: Mathematics Department Math-254
Second Semester 1433-34 H Final Examination (Multiple Choice)
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Q. No.	1	2	3	4	5	6	7	8	9	10
a,b,c,d	b	c	a	c	d	a	d	c	b	a

Q. No.	11	12	13	14	15	16	17	18	19	20
a,b,c,d	d	b	c	a	d	d	a	b	c	d

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Q. No.	1	2	3	4	5	6	7	8	9	10
a,b,c,d	d	a	c	b	a	b	c	a	d	c

Q. No.	11	12	13	14	15	16	17	18	19	20
a,b,c,d	b	a	b	c	a	a	d	d	a	b

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Q. No.	1	2	3	4	5	6	7	8	9	10
a,b,c,d	c	b	b	d	b	d	a	b	c	d

Q. No.	11	12	13	14	15	16	17	18	19	20
a,b,c,d	c	d	a	b	c	c	b	a	b	c

Question 21: Given $f(x) = (x+1)\ln(x+1)$, and $x_0 = 1, x_1 = 1.5, x_2 = 2.5, x_3 = 3$.

(i) Find the best approximation of $3\ln 3$ using quadratic Newton's divided differences interpolation formula.

(ii) Compute an error bound. [(3 + 2) points]

Question 22: (i) Let $f(x) \in C^2[x_0, x_1]$ and $h = x_1 - x_0$, then show that

$$\int_{x_0}^{x_1} f(x)\,dx \approx \frac{h}{2}[f(x_0) + f(x_1)]$$

(ii) Use the best numerical integration rule to approximate $\int_0^2 (x+2)e^x\,dx$ when $h = 0.5$.

(iii) Compute an error bound for your approximation. [(2 + 2+ 1) points]