King Saud University: Mathematics Department M-254
Semester I 2nd Midterm Exam 1440-1441 H
Maximum Marks=25 Time Allowed: 90 Mins.

Questions : $(5+5+5+5+5)$

Q1: Compute the inverse A^{-1}, where

$$A = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & 3 \\ 4 & 1 & 2 \end{pmatrix},$$

by solving $AB = \mathbf{I}$, using Gauss elimination by partial pivoting where $B = A^{-1}$. Then use it to solve the system $A\mathbf{x} = [1, 1, 2]^T$.

Q2: Use LU decomposition by Crout's method to find the value(s) of α for which the following matrix

$$A = \begin{pmatrix} 1 & 1 & \alpha \\ 1 & \alpha & 1 \\ \alpha & 1 & 1 \end{pmatrix},$$

is singular. Compute the unique solution of the linear system $A\mathbf{x} = [1, 1, -2]^T$ by using the smallest positive integer value of α.

Q3: Consider the following linear system of equations

$$\begin{aligned} 4x_1 - x_2 + x_3 &= 12 \\ -x_1 + 3x_2 + x_3 &= 1 \\ x_1 + x_2 + 5x_3 &= -14 \end{aligned}$$

Use Jacobi iterative method to find the second approximation $\mathbf{x}^{(2)}$ when the initial solution is $\mathbf{x}^{(0)} = [4, 3, -3]^T$. Compute the error bound for the **relative error** using the approximate solution $\mathbf{x}^{(2)}$.

Q4: Let $f(x) = \dfrac{1}{x}$ be defined in the interval $[2, 4]$ and $x_0 = 2,\ x_1 = 2.5,\ x_2 = 4$. Compute the value of the unknown point $\eta \in (2, 4)$ in the error formula of quadratic Lagrange interpolating polynomial for the approximation of $f(3)$ using the given points x_0, x_1, x_2.

Q5: Consider $f(x) = \sin(x)$ defined on $[0, 1]$. If the approximation of $\sin(0.28)$ by sixth degree equally spaced Lagrange interpolating polynomial is 0.27635557, then compute the error bound and the absolute error for the approximation.