King Saud University: Mathematics Department Math-254
Second MidTerm Examination Semester I 1438-1439 H
Time: 90 mins. Max. Mark 25

Problem 1: Compute the inverse A^{-1}, where (6)

$$A = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & 3 \\ 4 & 1 & 2 \end{pmatrix},$$

by solving the system $AB = \mathbf{I}$, using Gauss elimination by partial pivoting where $B = A^{-1}$. Then use it to solve the system $A\mathbf{x} = [1, 1, 2]^T$.

Problem 2: Find all values of α that make the following matrix singular using the LU decomposition by Doolittle's method (6)

$$A = \begin{pmatrix} 1 & 2 & \alpha \\ 2 & 8 & 2\alpha \\ \alpha & 2\alpha & 9 \end{pmatrix}.$$

Use the smallest positive integer value of α to find the unique solution of the system $A\mathbf{x} = [1, 2, 3]^T$ using Doolittle's method.

Problem 3: Find the matrix form of the Jacobi method $\mathbf{x^{(k+1)}} = T_J\mathbf{x^{(k)}} + \mathbf{c_J}$, $k \geq 0$, of the following system (7)

$$\begin{array}{rcrcrcr} 6x_1 & - & 3x_2 & + & x_3 & = & 11 \\ x_1 & - & 7x_2 & + & x_3 & = & 10 \\ 2x_1 & + & x_2 & - & 8x_3 & = & -15 \end{array}$$

Then use it to find the second approximation $\mathbf{x^{(2)}}$ of the solution $\mathbf{x}$ using the initial approximation $\mathbf{x^{(0)}} = [0, 0, 0]^T$. Compute an error bound for the error $\|\mathbf{x} - \mathbf{x^{(10)}}\|$.

Problem 4: Let $f(x) = \dfrac{1}{x}$ be defined in the interval $[2, 4]$ and $x_0 = 2,\ x_1 = 2.5,\ x_2 = 4$. Compute the value of the unknown point η in the error formula of quadratic Lagrange interpolating polynomial for the approximation of $f(3)$ using the given points x_0, x_1, x_2. (6)