King Saud University: Mathematics Department Math-254
Second MidTerm Examination Semester II 1438-1439 H
Time: 120 min. Max. Marks 25

Question 1: (6)

Use the simple Gaussian elimination method to find all values of α and β for which the following linear system is consistent or inconsistent. Also, find the solution of the consistent system.

$$\begin{aligned} 2x_1 &- x_2 + 3x_3 = 1 \\ 4x_1 &+ 2x_2 + 2x_3 = 2\alpha \\ 2x_1 &+ x_2 + x_3 = \beta \end{aligned}$$

Question 2: (7)

Consider the following matrix and its inverse as follows:

$$A = \begin{pmatrix} 10 & -1 & 0 \\ -1 & 10 & -2 \\ 0 & -2 & 10 \end{pmatrix} \quad \text{and} \quad A^{-1} = \begin{pmatrix} 48/475 & 1/95 & 1/475 \\ 1/95 & 2/19 & 2/95 \\ 1/475 & 2/95 & 99/950 \end{pmatrix}.$$

Show that Gauss-Seidel method converges faster than Jacobi method for the linear system $A\mathbf{x} = [9, 7, 6]^T$. If an approximate solution for this system is $x^* = [0.97, 0.91, 0.74]^T$, then find the relative error.

Question 3: (6)

Find α for which the following matrix A is singular using LU decomposition by Doolittle's method ($l_{ii} = 1$).

$$A = \begin{bmatrix} 1 & 2 & \alpha \\ 2 & 7 & 3\alpha \\ \alpha & 3\alpha & 4 \end{bmatrix}.$$

Then use the smallest positive integer value of α to find the solution of the linear system $A\mathbf{x} = [1, 0, 3]^T$ using Doolittle's method.

Question 4: (6)

Let $f(x) = \dfrac{1}{x}$ be defined in the interval $[2, 4]$ and $x_0 = 2,\ x_1 = 2.5,\ x_2 = 4$. Compute the value of the unknown point $\eta \in (2, 4)$ in the error formula of the quadratic Lagrange interpolating polynomial for the approximation of $f(3)$ using the given points. Also, compute an error bound for the corresponding error.