King Saud University

Math. Dept. M-254 Summer Semester 1439-40H
Second Midterm Exam Time: 90 mins. Max. Marks: 25

Questions: (5+5+5+5+5)

(1) Find the matrix form of Jacobi method $\mathbf{x}^{(k+1)} = T_J\mathbf{x}^{(k)} + \mathbf{c}_J$, $k = 0, 1, \ldots$ to the following system

$$\begin{aligned} 6x_1 &- 3x_2 + x_3 = 11 \\ x_1 &- 7x_2 + x_3 = 10 \\ 2x_1 &+ x_2 - 8x_3 = -15 \end{aligned}$$

If the initial approximation is $\mathbf{x}^{(0)} = [0, 0, 0]^T$, then compute error bound $\|\mathbf{x} - \mathbf{x}^{(5)}\|$.

(2) Find the condition number of the following matrix (for $n = 2, 3, \ldots$)

$$A_n = \begin{bmatrix} 1 & 1 \\ 1 & 1 - 1/n \end{bmatrix}.$$

If $n = 2$ and $x^* = [-1.99, 2.99]^T$ be the approximate solution of the linear system $A\mathbf{x} = [1, -0.5]^T$, then find the relative error.

(3) Let $p_2(x)$ be the quadratic Lagrange interpolating polynomial $p_2(x)$ for the data: $(1, 2), (2, 3), (3, \alpha)$. Find a value of α if the coefficient of x^2 in $p_2(x)$ is 2. Find the approximation of $f(2.5)$.

(4) Construct the table for $(\alpha, M(\alpha))$ by evaluating the integral at $\alpha = 1, 3, 5, 6$.

$$M(\alpha) = \int_0^1 (\alpha - e^x)\, dx.$$

Use the constructed table to find the best approximation of $M(4)$ by using quadratic Lagrange polynomial. Compute the absolute error.

(5) Consider the following table of date points

x	3	1	5	6
$f(x)$	1	−3	2	4

Find the third divided difference $f[3, 1, 5, 6]$ and use it to find Newton's (degree 3) form of the interpolating polynomial. Find approximation of $f(2)$.