King Saud University: Mathematics Department MATH-254
Summer Semester 1432-33 H Final Examination
Maximum Marks = 50 Time: 180 MIns.

Name of the Student:———————————— I.D. No. ————————

Name of the Teacher:———————————— Section No. ————————

The Answer Tables for Q.1 to Q.20 : Marks: 1.5 for each one ($1.5 \times 20 = 30$)

Note: Total number of questions are 24 and total number of pages are 8.

Ps. : Mark {a, b, c or d} for the correct answer in the box.

Q. No.	1	2	3	4	5	6	7	8	9	10
a,b,c,d										

Q. No.	11	12	13	14	15	16	17	18	19	20
a,b,c,d										

Quest. No.	Marks
Q. 1 to Q. 20	
Q. 21	
Q. 22	
Q. 23	
Q. 24	
Total	

Question 1: The error bound for the 5^{th} approximation to the solution of the nonlinear equation $f(x) = 0$ in $[1.5, 2]$ using bisection method is:

(a) $\frac{1}{8}$ (b) $\frac{1}{32}$ (c) $\frac{1}{64}$ (d) $\frac{1}{16}$

Question 2: If the root of the nonlinear equation $f(x) = 0$ in $[0.5, 2]$ is a fixed point of the equation $g(x) = \sqrt{2-x}$, then $f(x) = 0$ is:

(a) $x^2 + x - 2 = 0$ (b) $\frac{x}{\sqrt{2-x}} - x = 0$ (c) $\frac{\sqrt{2-x}}{x} - x = 0$ (d) $x^2 - x + 2 = 0$

Question 3: The iterative scheme $x_{n+1} = \frac{1}{2}(x_n + \frac{2}{x_n})$ converges to:

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 2 (d) $\sqrt{2}$

Question 4: Newton's formula for the approximation of $\sqrt[3]{a}$ is:

(a) $x_{n+1} = \frac{2x_n^3 + a}{3x_n^2}$ (b) $x_{n+1} = \frac{2x_n^3}{a + 3x_n^2}$ (c) $x_{n+1} = \frac{x_n + a}{3x_n^2}$ (d) $x_{n+1} = \frac{x_n^2}{x_n + a}$

Question 5: Given $x_0 = 2$ and $x_1 = 3$, then the next approximation x_2 of the solution of the equation $x^3 = 2x + 5$ using the Secant method is:

(a) 1.2 (b) 2.1 (c) 1.5 (d) 2.5

Question 6: The multiplicity of the root $\alpha = 1$ of the equation $x^4 - x^3 - 3x^2 + 5x = 2$ is:

(a) $m = 4$ (b) $m = 2$ (c) $m = 1$ (d) $m = 3$

Question 7: The goal of forward elimination step in simple Gauss elimination is to reduce the coefficient matrix to a matrix of the form:

(a) Upper-triangular (b) Lower-triangular (c) Identity (d) Diagonal

Note: The following information will be used in Questions 8 to 10:

$$A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}, \quad A^{-1} = \begin{bmatrix} 0.3 & -0.2 \\ -0.1 & 0.4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}.$$

Question 8: The solution of the linear system $A\mathbf{x} = \mathbf{b}$ using LU-decomposition ($l_{ii} = 1$) is:

(a) $[1.1, 0.7]^T$ (b) $[0.1, -0.7]^T$ (c) $[1.1, -0.7]^T$ (d) $[-1.1, -0.7]^T$

Question 9: The relative error with respect to the approximate solution $\hat{\mathbf{x}} = [0.4, -0.6]$ for l_∞-norm is bounded by:

(a) 2.7 (b) 2.6 (c) 2.8 (d) 2.9

Question 10: Using Jacobi iteration method with the initial approximation $[0, 0]^T$, the error bound $\|\mathbf{x} - \mathbf{x}^{(4)}\|$ is:

(a) $\frac{3}{32}$ (b) $\frac{3}{22}$ (c) $\frac{3}{26}$ (d) $\frac{3}{16}$

Question 11: If $log_{10}3 = 0.4771$ and $log_{10}4 = 0.6021$, then the approximation of $log_{10}3.4$ using linear Lagrange polynomial is:

(a) 0.5171 (b) 0.5371 (c) 0.5471 (d) 0.5271

Question 12: If the best approximation of $f(1.5)$ using Newton's quadratic interpolating polynomial is 7 and $f[1, 2, 3, 4] = 8$, then the Newton's cubic polynomial $p_3(1.5)$ gives:

(a) 9.0 (b) 10.0 (c) 12.0 (d) 11.0

Question 13: Using linear spline, which interpolates the data: $(1, 3), (2, 4), (3, 3), (4, 9)$, the interpolated value at $x = 2.3$ is:

(a) 6.5 (b) 4.5 (c) 3.7 (d) 4.7

Question 14: The number of subintervals required to approximate $\int_0^{0.2} \frac{1}{x+1}\,dx$ within the accuracy 10^{-3} by using Composite Trapezoidal's is:

(a) 2 (b) 3 (c) 1 (d) 4

Question 15: The highest degree of polynomial integrand for which composite Simpson's rule of integration is exact is:

(a) First degree (b) Second degree (c) Third degree (d) Fourth degree

Question 16: Using data points: $f(0.1) = 0.5, f(0.5) = 0.9, f(0.9) = 1, f(1.1) = 3, f(1.5) = 5$, the best approximation of $f'(1)$ using 3-point formula is:

(a) 20.0 (b) 10.0 (c) 30.0 (d) 15.0

Question 17: If $f(x) = x \ln x$ and $x_0 = 1, x_1 = 1.5, x_2 = 2$, then the error bound for the approximation of $f'(1.5)$ using three-point central formula is:

(a) 0.0833 (b) 0.0733 (c) 0.0633 (d) 0.0533

Question 18: If $f(2.20) = 0.5, f(2.25) = 1.5, f(2.70) = 2, f(2.75) = 3, f(3.25) = 6$, and $f(4) = 8$, then the best approximation of $f''(2.75)$ is:

(a) 6.5 (b) 5.75 (c) 6.0 (d) 3.25

Question 19: Given $y' + y = x + 1,\ y(0) = 1$, the approximate value of $y(0.1)$ using Euler's method when $n = 1$ is:

(a) 1.5 (b) 1.0 (c) 1.1 (d) 1.2

Question 20: Given $y' + y = x^2,\ y(0) = 0.5$, the approximate value of $y(0.2)$ using Runge-Kutta method of order two when $n = 1$ is:

(a) 0.414 (b) 0.144 (c) 0.441 (d) 0.041

Question 21: Find the first approximation $(x_1, y_1)^T$ of the following nonlinear system

$$\begin{aligned} x^2 + y^2 &= 4 \\ 2x - y^2 &= 0 \end{aligned}$$

using Newton's method, starting with $(x_0, y_0)^T = (1, 1)^T$. [5 points]

Question 22: Show that for the nonhomogeneous linear system $A\mathbf{x} = \mathbf{b}$, with the matrix A

$$A = \begin{pmatrix} 5 & 0 & -1 \\ -1 & 3 & 0 \\ 0 & -1 & 4 \end{pmatrix}$$

Gauss-Seidel iterative method converges faster than **Jacobi iterative method.**[5 points]

Question 23: Consider the points $x_0 = 0.5, x_1 = 1.5, x_2 = 2.5, x_3 = 3.0, x_4 = 4.5$ and for a function $f(x)$, the divided differences are:

$f[x_2] = 73.8125, f[x_1, x_2] = 59.5, f[x_0, x_1, x_2] = 23.7, f[x_1, x_2, x_3] = 47.25, f[x_0, x_1, x_2, x_3, x_4] = 1.$

Use these information, construct the **complete divided differences table** for the given data points. [5 points]

<u>**Question 24:**</u> Let $x_0 \in (a,b)$, where $f \in C^3[a,b]$ and that $x_1 = x_0 + h \in (a,b)$, $x_2 = x_0 + 2h \in (a,b)$ for some $h \neq 0$, then show that

$$f^{(1)}(x_0) \approx \frac{-3f(x_0) + 4f(x_0+h) - f(x_0+2h)}{2h}.$$

Use the **two-point formula** to find the approximate value of the derivative $f'(2.5)$ of the function $f(x) = (x+1)\ln(x+1)$, with $h = 0.05$. Also, compute an error bound. [5 points]