King Saud University-College of Science-Department of Mathematics

• **Final Exam (Time: 3 hours)-First Semester 1438-1439**

December 27, 2017

EXERCICE1:

1- Determine $\sup\{x \in \mathbb{R}, \quad x^4 - 3x^2 + 2 < 0\}$ and $\inf\{x \in \mathbb{R}, \quad x^4 - 3x^2 + 2 > 0\}$.

2- Find:

$$\lim_{x \mapsto +\infty} (1 + \frac{3}{x})^{2x} \quad \text{and} \quad \lim_{x \mapsto +\infty} \sqrt{x} - \sqrt{x + \sqrt{x - \sqrt{x}}}.$$

3- Using the definition prove that $\lim_{x \mapsto 0} \frac{x^2-1}{x^2+1} = -1$ and $\lim_{x \mapsto +\infty} x^2 - 1 = +\infty$.

EXERCICE2:

1- Study the convergence of the following series:

$$\sum_{n \geq 0} \frac{n+1}{n^3+1}, \quad \sum_{n \geq 0} (-1)^n \frac{n^2}{n^3+1}, \sum_{n \geq 1} \frac{2^n}{n!} \quad \text{and} \quad \sum_{n \geq 1} \frac{n}{3^n}.$$

2- Study the convergence of the following integrals:

$$\int_0^1 \frac{dx}{x(x^2+1)}, \int_0^1 \frac{dx}{\sqrt{x(1-x)}}, \int_2^\infty \frac{dx}{xlnx} \quad \text{and} \quad \int_1^\infty \frac{dx}{x^2+x+1}.$$

EXERCICE3:

1- Let $f : [-1, 1] \to [-1, 1]$ be a continuous function such that $f(x) + f(-x) = 0$. Prove that there exist $2n + 1$ real $c \in [-1, 1]$ such that $f(c) = c$.

2- Prove that for all $x > y > 0$, we have $1 - \frac{y}{x} < \ln(x) - \ln(y) < \frac{x}{y} - 1$. Deduce that

$$\frac{1}{y+1} < \ln(1 + \frac{1}{y}) < \frac{1}{y}.$$

EXERCICE4:

Let $\{f_n\}$ be the sequence of functions on $(0, +\infty)$ defined by $f_n(x) = \frac{nx}{1+n^2x^2}$.

1- Prove that this seqúece converges pointwise to zero.

2- Prove that this sequece is not uniformly convergent.