King Saud University

Department of Mathematics

Final Exam 280-Math 1Semester (1439/1440)

Question 1(6°). a) Show that the set $E=\{cn,\ n\in\mathbb{N}\},\ c\in\mathbb{R}^{+}$ is not bounded.

b) Show that the set $E=\{\dfrac{2^n n!}{n^n},\ n\in\mathbb{N}\}$ is bounded.

Question 2(6°). Determine whether each of the following series is convergent or divergent:

a) $\sum_{n=1}^{\infty}\dfrac{(-1)^n n}{e^{n^2+1}}$, b) $\sum_{n=1}^{\infty}\sqrt[n]{e^n+\pi^n}$

Question 3 (4°). Show that $1-\cos x\le x\ \ \forall x\ge 0\ \ and\ \ x\le 1-\cos x\ \ \forall x\le 0$

Question 4 (6°). a) Calculate the limit: $\lim_{x\to 0}\dfrac{e^{\sin x}-1-\sin x-\frac{1}{2}\sin^2 x}{\sin^3 x}$

b) Determine whether the following improper integral converges or diverges:

$$\int_1^{\infty}\frac{x}{2+x^2+\frac{1}{x^2}}\,dx$$

Question 5 (6°). Calculate the following limit: $\lim_{n\to\infty}\left(\int_0^1\dfrac{nx^2+\sin nx^2}{n}\,dx+\sum_{k=1}^{n}\dfrac{k}{4n^2+k^2}\right)$

Question 6 (6°). a) Show that the series $\sum_{n=1}^{\infty}\dfrac{\sin nx}{n^3}$ is convergent for any $x\in\mathbb{R}$.

b) Show that the function $\sum_{n=1}^{\infty}\dfrac{\sin nx}{n^3}$ is continuous on $\mathbb{R}$.

c) Show that if $F(x)=\sum_{n=1}^{\infty}\dfrac{\sin nx}{n^3}$ then $F'(x)=\sum_{n=1}^{\infty}\dfrac{\cos nx}{n^2}$.

Question 7 (6°). (a) Find the sum of the function series $\sum_{n=0}^{\infty}\dfrac{(-1)^n x^{n+1}}{n+1}$

(b) Show that the number series $\sum_{n=1}^{\infty}\dfrac{(-1)^{n-1}}{n2^n}$ converges and $\sum_{n=1}^{\infty}\dfrac{(-1)^{n-1}}{n2^n}=\ln 3-\ln 2$