KING SAUD UNIVERSITY, COLLEGE OF SCIENCE, DEPARTMENT OF MATHEMATICS: MATHS-280.

FINAL EXAM (TIME: 3 HOURS), SECOND SEMESTER, 1436-1437H.

EXERCICE1:

1- Determine the following infimum:

$$inf\{z = x + \frac{1}{x}, \quad x > 0\}.$$

2- For $a \geq 0$ and $b \geq 0$, find the limit of the sequence:

$$\lim_{n \mapsto +\infty} n - \sqrt{n+a}\sqrt{n+b}.$$

3- Study the convergence of the following series:

$$\sum_{n=0}^{n=+\infty} \frac{n}{n^2+n+1} \quad \text{and} \quad \sum_{n=1}^{n=+\infty} \frac{n}{e^n}.$$

EXERCICE2:

1- Using the $\varepsilon - \delta$ definition of the limit, show that

$$\lim_{x \mapsto 0} \frac{x}{1+\cos^2 x} = 0.$$

2- Find the following limit:

$$\lim_{x \mapsto +\infty} x ln(1 + \frac{2}{x}).$$

3- Find The local extrema of the function $f(x) = 5x^5 - 20x^3$.

4- Show that $|\cos(2a) - \cos(2b)| \leq 2|a-b|$, for all real numbers a and b.

EXERCICE3:

1- Show that if f is a function Riemannian integrable on $[0, 1]$, then

$$\lim_{n \mapsto +\infty} \frac{1}{n} \sum_{k=1}^{n} f(\frac{k}{n}) = \int_0^1 f(x)dx.$$

2- Use (1) to calculate the limit:

$$\lim_{n \mapsto +\infty} \sum_{k=1}^{n} \frac{n}{n^2+k^2}.$$

3- Test the convergence of the improper integrals:

$$\int_0^1 \frac{1}{x\sqrt{x-1}} \quad \text{and} \quad \int_1^{+\infty} \frac{x}{x^3+1}.$$

EXERCICE4:

1- Find the pointwise limit of the sequence of functions:

$$f_n(x) = \frac{1}{1+(nx-1)^2}, \quad n \in \mathbb{N}, x \in [0, 1].$$

2- Study the uniform convergence of the sequence of functions $(f_n(x))$.