King Saud University-College of Science-Department of Mathematics

Final Exam (Time: 3 hours)-Second Semester 1437-1438

May 18, 2017

EXERCICE1:

1- Prove that $\sqrt{5}$ is irrational.

2- Determine $\sup\{x \in \mathbb{R}, \quad x^3 + 4x^2 + 3x < 0\}$ and $\inf\{x \in \mathbb{R}, \quad x^3 + 4x^2 + 3x > 0\}$.

3- For $m > 0$ and $n > 0$, find:

$$\lim_{x \mapsto 0} \frac{\sqrt{1+x^n} - \sqrt{1-x^n}}{x^m} \qquad \lim_{x \mapsto +\infty} (1 - \frac{2}{x})^{3x} \quad \text{and} \quad \lim_{x \mapsto +\infty} \sqrt{x + \sqrt{x + \sqrt{x}}} - \sqrt{x}.$$

EXERCICE2:

1- Using the definition prove that $\lim_{n \mapsto +\infty} \frac{n^2-1}{n^2+1} = 1$.

2- Find the following sum:

$$\sum_{n \geq 0} \sqrt{n+1} - \sqrt{n} \quad \text{and} \quad \sum_{n \geq 5} (\frac{-1}{3})^n.$$

3- Study the convergence of the following series:

$$\sum_{n \geq 0} \frac{n}{n^3+1}, \sum_{n \geq 0} \frac{n^2}{n^3+1}, \sum_{n \geq 1} \frac{2^n}{n+1} \quad \text{and} \quad \sum_{n \geq 1} \frac{(-1)^n}{1+\sqrt{n}}.$$

EXERCICE3:

1- Using the definition prove that: $\lim_{x \mapsto 1} x^2 + 1 = 2$.

2- For $f : [0, 2] \to [0, 2]$ be a continuous function. Prove that there exist $c \in (0, 2)$ such that $f(c) = c$.

3- Prove that for all $x > y > 0$, we have $1 - \frac{y}{x} < \ln(x) - \ln(y) < \frac{x}{y} - 1$.