King Saud University

Department of Mathematics

Final Exam **280-Math** **2 Semester (1438/1439)**

Question1(4°). Determine the sup, max, inf and min of the set $E=\{\frac{m+m^2n+n+n^2m}{nm};\ m,n\in \mathrm{N}\}$

Question2(4°). Determine whether each of the two following series is absolutely convergent, conditionally convergent or divergent: $A=\sum_{n=1}^{\infty}\frac{2n+1}{\sqrt{n2^n}}$. $B=\sum_{n=1}^{\infty}(-1)^n(2+\frac{1}{n})^n$

Question3(6°). (a) Show that the sequence $x_n=\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\cdots+\frac{1}{2n}$ is bounded.

(b) Calculate the $\lim_{n\to\infty} x_n$.

Question4(5°). (a) Using the $(\varepsilon-\delta)$ definition show that $\lim_{x\to 0}\frac{5x}{1+x^2+\cos^2 x}=0$

(b) Find the following limit: $\lim_{x\to 0}\frac{\sin x^4-x^4+\frac{1}{6}x^{12}}{x^{20}}$.

Question5(3°). show that if $f:[a,b]\to[a,b]$ is a continuous function, then $\exists\ c\in[a,b]$ such that $f(c)=c$.

Question6(4°). Decide whether the improper integral $\int_1^{\infty}\frac{x}{1+x^2\sin^2 x}dx$ is convergent or divergent.

Question7(7°). (a) Study the uniform convergence of the sequence $f_n(x)=e^{-nx^2}(x^2+1)+\frac{n^2x+1}{n^2+1}$

(1) on $\Re$ (2) on the interval $[1,2]$.

(b) Evaluate the limit: $\lim_{n\to\infty}\int_1^2\left(e^{-nx^2}(x^2+1)+\frac{n^2x+1}{n^2+1}\right)dx$

Question8(7°). (a) Find the sum of the function series $\sum_{n=0}^{\infty}(n+1)x^n$

(b) Find the sum of the number series $\sum_{n=0}^{\infty}\left(\frac{2^n}{3^n}n+(\frac{2}{3})^n\right)$