King Saud University

Department of Mathematics

**Final Exam** **280-Math** **2Semester (1439/1440)**

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**Question1**( $5^\circ$ ). let $E$ be a bounded set. Show that: (a) if $E$ has a max, then its unique.

(b) $\inf E$ is unique.

**Question2**( $6^\circ$ ). (a) Decide whether the set $E_n = \{\sqrt[n]{\alpha^n + (\alpha+\beta)^n}\}$, $\alpha > 0, \beta > 0$ is bounded.

(b) Find $\lim_{n\to\infty} x_n$ if $x_n = 2^{(-1)^n - n}$ .

**Question3**( $4^\circ$ ). Determine whether each of the following series is convergent or divergent:

(a) $\sum_{n=1}^{\infty} \sin\frac{1}{n}$ , (b) $\sum_{n=1}^{\infty} n\sin\frac{1}{n^2}$, (c) $\sum_{n=1}^{\infty} \cos\frac{1}{n}$ , (d) $\sum_{n=1}^{\infty} \cos\frac{1}{n^2}$

**Question4** ( $4^\circ$ ). Using the $(\varepsilon - \delta)$ definition of the limit, show that $\lim_{x\to 0} \dfrac{2x\sin\frac{1}{x}}{1+\tan^2 x} = 0$

**Question5**( $5^\circ$ ). Show that the improper integral $\int_0^{\pi^2/4} \frac{\sin x}{\sqrt{x^3}}dx$ is convergent and its value $\le \pi$.

**Question6** ( $5^\circ$ ). Find $\lim_{n\to\infty} x_n$ of the sequence $x_n = \int_1^2 (e^{-nx^2} + x)\,dx$ .

**Question7**( $5^\circ$ ). Represent the function $\int_0^x \frac{\sin 3x}{x}dx$ by power series of the form $\sum_{n=0}^{\infty} a_n x^n$ .

**Question8**( $6^\circ$ ).(a) Find the sum of the function series $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2}}{n+1}$

(b) Find the sum of the number series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n4^n}$ .

(c) Find the sum of the number series $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$