King Saud University

Department of Mathematics

Final Exam | **280-Math** | **2Semester (1439/1440)**

Question1(5°). let E be a bounded set. Show that: (a) if E has a max, then its unique.

(b) $\inf E$ is unique.

Question2(6°). (a) Decide whether the set $E_n = \{\sqrt[n]{\alpha^n + (\alpha+\beta)^n}\}$, $\alpha > 0, \beta > 0$ is bounded.

(b) Find $\lim\limits_{n\to\infty} x_n$ if $x_n = 2^{(-1)^n - n}$

Question3(4°). Determine whether each of the following series is convergent or divergent:

(a) $\sum\limits_{n=1}^{\infty} \sin\frac{1}{n}$, (b) $\sum\limits_{n=1}^{\infty} n\sin\frac{1}{n^2}$, (c) $\sum\limits_{n=1}^{\infty} \cos\frac{1}{n}$, (d) $\sum\limits_{n=1}^{\infty} \cos\frac{1}{n^2}$

Question4 (4°). Using the $(\varepsilon - \delta)$ definition of the limit, show that $\lim\limits_{x\to 0} \dfrac{2x\sin\frac{1}{x}}{1+\tan^2 x} = 0$

Question5(5°). Show that the improper integral $\int_0^{\pi^2/4} \dfrac{\sin x}{\sqrt{x^3}}\,dx$ is convergent and its value $\le \pi$.

Question6 (5°). Find $\lim\limits_{n\to\infty} x_n$ of the sequence $x_n = \int_1^2 (e^{-nx^2} + x)\,dx$.

Question7(5°). Represent the function $\int_0^x \dfrac{\sin 3x}{x}\,dx$ by power series of the form $\sum\limits_{n=0}^{\infty} a_n x^n$.

Question8(6°).(a) Find the sum of the function series $\sum\limits_{n=0}^{\infty} \dfrac{(-1)^n x^{2n+2}}{n+1}$

(b) Find the sum of the number series $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^{n-1}}{n4^n}$.

(c) Find the sum of the number series $\sum\limits_{n=0}^{\infty} \dfrac{(-1)^n}{n+1}$