King Saud University
Department of Mathematics

Final Exam 280-Math
Summer Semester (1437/1438)

Question1 (4). Determine the sup, max, inf and min of the set $E = \left\{1 + \frac{(-1)^n}{n}\ , n \in N\right\}$.

Question2 (4). Decide whether the following sequences are Cauchy:

$$a_n = \frac{(-1)^n n}{2n+1} \quad ; \quad b_n = (-1)^n + \frac{1}{n}$$

Question3 (4). Decide whether the following sequence is convergent or divergent:

$$x_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n}$$

Question4 (8). Test the following series:

(A) $\sum_{n=1}^{\infty} \frac{\pi}{n^2} \sin\frac{\pi}{n}$ (B) $\sum_{n=1}^{\infty} \frac{n}{\pi} \sin\frac{\pi}{n}$ (C) $\sum_{n=1}^{\infty} \left(\frac{5}{4} + \frac{\sin 1/n}{n}\right)^n$ (D) $\sum_{n=1}^{\infty} \frac{4^n n!}{n^n}$

Question5 (4). Assuming that the function $f(x) = \begin{cases} \frac{(x^2-5x+4)\sin(x-4)}{(x-4)^2} & , \ x \neq 4 \\ 2x + \beta & , \ x = 4 \end{cases}$

is continuous at $x = 4$, find the value of the number β.

Question6 (6). (a) Show that the equation $\cos x = x$ has a solution in $\left(0, \frac{\pi}{2}\right)$.

(b) Find the extrema of $f(x) = x^3 - x$ on $[-1, 2]$.

Question7 (4). Let $f_n : [1,2] \to R$ such that $f_n(x) = \frac{x}{(1+x)^n}$.

Show that the series $\sum_{n=1}^{\infty} f_n(x)$ is uniformly convergent.

Question8 (6). (a) Discuss the convergence of the following improper integral

$$\int_1^{\infty} \frac{dx}{\sqrt{1+x^3}}$$

(b) Find the radius of convergence and the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{n+1}{10^n} (x-4)^n$.