King Saud University Department of Mathematics

Mid Term Exam 280-Math 1Semester (1443)H

Question 1 [3+3]

Use induction to prove:

a. If $x > -1$, prove that $(1+x)^n \geq 1 + nx$, for all $n \in \mathbb{N}$,.
b. Prove that $2^{n-1} \leq n!$, for all $n \in \mathbb{N}$,.

Question 2 [3+3]

Determine sup A and Inf A where they exist:

1. $A = \left\{n \in \mathbb{N}, \ \frac{n+(-1)^n}{n+1}\right\}$,
2. $A = \{x \in \mathbb{R}, \ |x| + |x-1| \leq 1\}$.

Question 3 [1+2+2+2]

Determine whether the sequence (x_n) is convergent or divergent, and find the limit where it exists:

a. $x_n = \frac{\sin(n)}{n}$,
b. $x_n = \frac{n+\sin(n)}{3^n}$
c. $x_n = \frac{2^n - 3^n}{2^n + 3^n}$
d. $x_{n+1} = \frac{4n^2-1}{4n^2} x_n, \quad x_1 = 1$

Question 4 [2+2+2]

Test the following series for convergence:

a. $\sum_{n=1}^{\infty} \frac{3^n}{n!}$
b. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$
c. $\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln(n))^n}$