

EXERCISE: 1

1 - Determine the supremum and the infimum (and justify your answer) of the following set : $A = \left\{ \frac{m}{2m+3n}, m, n \in \mathbb{N} \right\}$

2 - Find $\sup\{ x \in \mathbb{R}, x^2 - x - 2 < 0 \}$.

3 - Prove that $\sqrt{3}$ is irrational.

4 - Find $\limsup_{n \rightarrow +\infty} \left\{ 2 + \frac{(-1)^n}{3n} \right\}$ and $\liminf_{n \rightarrow +\infty} \left\{ 2 + \frac{(-1)^n}{3n} \right\}$

EXERCISE: 2

1 - Using the definition find the limit of the sequence $\frac{3^n - 1}{2^{n+1}}$.

2 - We consider the sequence $x_n = \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n}$. Prove that this sequence is increasing, convergent and its limit belongs to $\left[\frac{1}{2}, 1 \right]$.

3 - Prove that $y_n = 1 + \frac{1}{2} + \dots + \frac{1}{n}$ is not a Cauchy sequence!

EXERCISE: 3

1 - Find the sum of the series: $\sum_{n=1}^{+\infty} \frac{1}{n(n+1)}$ and $\sum_{n=4}^{+\infty} (2^n + (-3)^{3n})$

2 - Test the convergence of the series: $\sum_{n=1}^{+\infty} \frac{n + \sin(n)}{n + \cos(n)}$ and $\sum_{n=1}^{+\infty} \frac{n^{p+1}}{n^{q+1}}$