King Saud University-College of Science-Department of Mathematics

First Mid Term Exam (Time: 1h30m)-Second Semester 1437-1438

March 7, 2017

EXERCICE1:

1- Determine the following

$$\sup\{x \in \mathbb{R}, \quad x^2 + 2x - 15 < 0\} \quad \text{and} \quad \inf\{x \in \mathbb{R}, \quad x^2 + 2x - 15 < 0\}.$$

2- Let $x_n = (-1)^n + \frac{2}{n^2}$, Find

$$\limsup_{n \mapsto +\infty} x_n \quad \text{and} \quad \liminf_{n \mapsto +\infty} x_n.$$

3- Prove that $\sqrt{3}$ is irrational.

EXERCICE2:

1- Using the definition of convergence to prove that

$$\lim_{n \mapsto +\infty} \frac{2n+1}{n+1} = 2 \quad \text{and} \quad \lim_{n \mapsto +\infty} e^n + 1 = +\infty.$$

2- Let $a > b > 0$, Find the following limits:

$$\lim_{n \mapsto +\infty} \frac{2a^n - 3b^n}{a^n + 2b^n} \quad \text{and} \quad \lim_{n \mapsto +\infty} (\sqrt{2a^n - b^n})^{\frac{2}{n}}.$$

EXERCICE3: Let $x_0 = 1$, and for all $n \geq 1$, $x_{n+1} = 2x_n + 1 - n$.

1- Find x_1, x_2 and x_3.

2- Prove that $n \leq x_n$, and deduce the limit of the sequence (x_n).

EXERCICE4:

1- Prove that every convergent sequence is a Cauchy sequence.

2- We define the sequence $x_n = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}$.

3- Prove that $x_{2n} - x_n \geq \frac{1}{2}$.

4- The sequence x_n is convergent?.