

Question1(3+3). Determine the sup, max, inf and min of the following sets:

$$F = \{2x^2 - 16x + 23, x \in \mathbb{R}\}$$

$$E = \left\{ \frac{n^2 + 9m^4 + 18nm^2}{3m^2n}, m, n \in \mathbb{N} \right\}$$

Question2 (2+2).

a) Using the Definition show that $\lim_{n \rightarrow \infty} \frac{3n^2 + 1}{2n^2} = \frac{3}{2}$

b) Show that the sequence $x_n = \frac{2^n n!}{n^n}$ is bounded above.

Question3 (3+3). Decide whether the following sequences are convergent or divergent with finding the limit in case of convergence:

a) $x_n = \frac{4^n n!}{(2n)!}$

b) $x_n = \frac{e^n + \sin n + (-1)^n + 3^n}{3^n}$

Question4 (3+2+2+2). Determine whether the following series are convergent or divergent:

a) $\sum_{n=1}^{\infty} \sqrt[n]{2^n n^{n+1}} \sin \frac{1}{n}$

b) $\sum_{n=0}^{\infty} \frac{\sqrt{9n^3 + 2n + 1}}{n^2 + 5}$

c) $\sum_{n=1}^{\infty} \sin \frac{n + 2\pi}{2n + \pi}$

c) $\sum_{n=1}^{\infty} \sin \frac{n + 10}{n^2}$