KING SAUD UNIVERSITY, COLLEGE OF SCIENCE, DEPARTMENT OF MATHEMATICS: MATHS-280.

SCOND MIDTERM EXAM (TIME: 90 MINUTES), FIRST SEMESTER, 1438-1439H.

**Exercise1:**

1- Using the definition prove that

$$\lim_{x\mapsto 1} x^2+1=2.$$

2- Find the following limits:

$$\lim_{x\mapsto 1}\frac{x^3-1}{x^2-1} \quad \text{and} \quad \lim_{x\mapsto +\infty} x(\sqrt{x^2+1}-x).$$

**Exercise2:**

1- Find the constant $a$ such that the function $f$ is continuous:

$$f(x)=\begin{cases} \dfrac{e^x-1}{x}+1, & x=0, \\ a, & x=0. \end{cases}$$

2- Prove that the function $g(x)=x^2$ is uniformly continous on $(0,1)$ but is not uniformly continous on $(0,+\infty)$.

**Exercise3:**

1- Show that the equation

$$\tan(x)-x=0$$

has at least three solutions in $[-\frac{\pi}{4},\frac{\pi}{4}]$.

2- Prove that for all $n\in\mathbb{N}$, we have

$$x^{2n+1}+x^n+1=0$$

has at least one real solutions

**Exercise4:** Study the convergence of the following series:

$$\sum_{n=0}^{\infty}\frac{1}{n^2+2},\quad \sum_{n=0}^{\infty}(-1)^n\frac{n+1}{n^2+1},\quad \sum_{n=0}^{\infty}\frac{e^n}{n^4+1} \quad \text{and} \quad \sum_{n=0}^{\infty}\frac{n}{3^n+1}.$$