KING SAUD UNIVERSITY, COLLEGE OF SCIENCE, DEPARTMENT OF MATHEMATICS: MATHS-280.

SCOND MIDTERM EXAM (TIME: 90 MINUTES), SECOND SEMESTER, 1436-1437H

EXERCICE1:

1- For a and b reels, study the convergence of the following series:

$$\sum_{n=1}^{+\infty} \frac{1}{1+n^a} \qquad \sum_{n=1}^{+\infty} \frac{(-1)^n}{n^a} \qquad \text{and} \qquad \sum_{n=1}^{+\infty} \frac{1}{n^b \ln(n)}$$

2- Find the following limits:

$$\lim_{x\to 1} \frac{x^2-1}{x-1}, \quad \lim_{x\to 0} \frac{x\sin(x)}{1-\cos(x)} \quad \text{and} \quad \lim_{x\to+\infty} x\left(\sqrt{x^2+1}-x\right).$$

EXERCICE2:

1- Find c such that the function f is continuous:

$$f(x) = \begin{cases} x^2+\frac{3x+2}{x-1}, & x \neq 1 \\ c, & x = 1 \end{cases}$$

2- Show that the equation

$$\sin(x) + \frac{1}{2} = x$$

has a solution in $(0, \frac{\pi}{2})$.

EXERCICE3:

1- Find The local extrema of the function

$$f(x) = x^4 - 4x^3 + 8x.$$

2- Prove that

$$\frac{1}{x+1} < \ln\left(\frac{x+1}{x}\right) < \frac{1}{x}$$

for all $x > 0$.