

King Saud University  
Department of Mathematics

2 Mid Term Exam

280-Math

2 Semester (1439/1440)

**Question1** (5). Find the  $\lim_{x \rightarrow 1} f(x)$  of the following functions or show that it does not exist:

(a)  $f(x) = \sin \frac{1}{x-1}$

(b)  $f(x) = (x-1) \sin \frac{1}{x-1}$

**Question2** (4+2\*). (a) Find the points of local maximum and local minimum of the function

$$f(x) = \frac{2(x^2 + 3)}{x^2 + 2x + 5}$$

(b)\* Sketch the graph of the function  $f(x)$ .

**Question3** (3). Show that  $\sin^{-1} x^2 + \cos^{-1} x^2 = \frac{\pi}{2}$ ,  $x \in [0,1]$

**Question4** (4). Show that if  $F(x) = x^6 - e^{\cos \frac{\pi}{2} x}$ , then

(a)  $\exists c_1 \in \mathbb{R}$  st  $F(c_1) = 0$       (b)  $\exists c_2 \in \mathbb{R}$  st  $F'(c_2) = 0$

**Question5** (3). Decide whether the function  $f(x) = \frac{x^2 + 2x + 1}{x^2 + 1}$  is uniformly continuous on  $\mathbb{R}$ .

**Question6** (3). Decide whether the integral  $\int_1^{\infty} \frac{1}{\sqrt{x} + \sqrt[3]{x}} dx$  is convergent or divergent.

**Question7** (3). Find  $\lim_{n \rightarrow \infty} x_n$  if  $x_n = \frac{1}{4n^2 + 1} + \frac{2}{4n^2 + 4} + \frac{3}{4n^2 + 9} + \dots + \frac{n}{5n^2}$