قسم الرياضيات	316 ريض	الفصل الثاني
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1.

$$a = \frac{\langle f, P_0 \rangle}{\|P_0\|} = \frac{1}{2}\int_{-1}^{1} e^x dx = \frac{1}{2} e^x \Big|_{-1}^{1} = \frac{1}{2}\left(e^1 - e^{-1}\right) = \sinh 1$$

$$b = \frac{\langle f, P_1 \rangle}{\|D_1\|} = \frac{3}{2}\int_{-1}^{1} e^x x dx = \frac{3}{2}\left[xe^x\Big|_{-1}^{1} - \int_{-1}^{1} e^x dx\right]$$

$$= \frac{3}{2}\left[e^1 + e^{-1} - 2\sinh 1\right]$$

$$= \frac{3}{2}\left[e^1 + e^{-1} - e^1 + e^{-1}\right] = 3e^{-1}$$

2.

(a)

$$a_0 = \frac{1}{4}\int_{-2}^{2} f(x) = \frac{1}{4}\cdot 4 = 1$$

$$a_n = \frac{1}{2}\int_0^2 2x \cos\frac{n\pi}{2} x\, dx$$

$$= x\frac{2}{n\pi}\sin\frac{n\pi}{2}x\Big|_0^2 - \frac{2}{n\pi}\int_0^2 \sin\frac{n\pi}{2}x = \left(\frac{2}{n\pi}\right)^2 \cos\frac{n\pi}{2}x\Big|_0^2$$

$$= \frac{-4}{n^2\pi^2}\left[1 - (-1)^n\right]$$

$$b_n = \frac{1}{2}\int_0^2 2x \sin\frac{n\pi}{2}x\, dx$$

$$= -x\frac{2}{n\pi}\cos\frac{n\pi}{2}x\Big|_0^2 + \frac{2}{n\pi}\int_0^2 \cos\frac{n\pi}{2}x\, dx = -\frac{4}{n\pi}(-1)^n$$

$$f(x) = 1 + \frac{8}{\pi^2}\sum_0^\infty \frac{1}{(2n+1)^2}\cos\frac{(2n+1)\pi}{2}x - \frac{4}{\pi}\sqrt{a^2 + b^2}\sum_1^\infty (-1)^n \sin\frac{n\pi}{2}x$$

(b)

not Uniformly convergent $\because\ f(-2) \neq f(2)$.

3.

(a)

$f(x,t) = e^{2xt-t^2}$ is analytic in both x, t.

$$\therefore\ f(x,t) = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{\partial^n f}{\partial t^n}\bigg|_{t=0} t^n$$

$$\begin{aligned}\frac{\partial^n f}{\partial t^n}\bigg|_{t=0} &= \frac{\partial^n}{\partial t^n} e^{x^2-(x-t)^2}\bigg|_{t=0} \\ &= e^{x^2} \frac{\partial^n}{\partial t^n} e^{-(x-t)^2}\bigg|_{t=0} \\ &= (-1)^n e^{x^2} \frac{d^n}{du^n} e^{-u^2}\bigg|_{u=x} \\ &= (-1)^n e^{x^2} \frac{d^n}{du^n} e^{-x^2} = H_n(x)\end{aligned}$$

(b)

$$c_2 L_2(x) + c_1 L_1(x) + c_0 L_0(x) = c_2\left(1 - 2x + \frac{1}{2}x^2\right) + c_1(1-x) + c_0 = x^2 + x + 1$$

$$\Rightarrow c_2 = 2,\ \ c_1 = -5,\ \ c_0 = 4$$

4.

$$\Gamma(x) = \int_0^{\infty} e^{-t} t^{x-1} dx$$

(a)

$$\begin{aligned}\Gamma(x+1) &= \int_0^{\infty} e^{t} t^{x} dt = -e^{-t} t^{x}\Big|_0^{\infty} + x\int_0^{\infty} e^{-t} t^{x-1} dt \\ &= 0 + x\Gamma(x)\end{aligned}$$

(b)

$$\Gamma(1) = \int_0^{\infty} e^{-t} dt = 1$$

$$\Gamma(n+1) = n\Gamma(n) = n(n-1)\Gamma(n-2) = \cdots = n(n-1)(n-2)\cdots(2)\Gamma(1) = n!$$

5.

$$J_\nu(x) = \left(\frac{x}{2}\right)^\nu \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\Gamma(\nu+m+1)} \left(\frac{x}{2}\right)^{2m}$$

$$J_\nu'(x) = \frac{\nu}{2}\left(\frac{x}{2}\right)^{\nu-1} \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\Gamma(\nu+m+1)} \left(\frac{x}{2}\right)^{2m} + \left(\frac{x}{2}\right)^\nu \sum_{m=1}^{\infty} \frac{(-1)^m m}{m!\Gamma(\nu+m+1)} \left(\frac{x}{2}\right)^{2m-1}$$

$$\begin{aligned} xJ_\nu'(x) &= \nu\left(\frac{x}{2}\right)^\nu \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\Gamma(\nu+m+1)} \left(\frac{x}{2}\right)^{2m} + x\left(\frac{x}{2}\right)^\nu \sum_{m=1}^{\infty} \frac{(-1)^m}{(m-1)!\Gamma(\nu+m+1)} \left(\frac{x}{2}\right)^{2m-1} \\ &= \nu J_\nu(x) - x\left(\frac{x}{2}\right)^\nu \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\Gamma(\nu+m+2)} \left(\frac{x}{2}\right)^{2m+1} \\ &= \nu J_\nu(x) - x\left(\frac{x}{2}\right)^{\nu+1} \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\Gamma(\nu+1+m+1)} \left(\frac{x}{2}\right)^{2m} \\ &= \nu J_\nu(x) - xJ_{\nu+1}(x) \end{aligned}$$

6.

even function

$$\therefore\ B(\xi) = 0$$

$$\begin{aligned} A(\xi) &= 2\int_0^{\pi/2} \cos x \cos \xi x \, dx \\ &= \int_0^{\pi/2} \left[\cos(1-\xi)x + \cos(1+\xi)x\right] dx \\ &= \frac{1}{1-\xi}\sin(1-\xi)\frac{\pi}{2} + \frac{1}{1+\xi}\sin(1+\xi)\frac{\pi}{2} \\ &= \frac{1}{1-\xi}\cos\frac{\pi}{2}\xi + \frac{1}{1+\xi}\cos\frac{\pi}{2}\xi = \frac{2\cos\frac{\pi}{2}\xi}{1-\xi^2} \end{aligned}$$

(b)

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} A(\xi)\cos \xi x \, dx = \frac{1}{\pi}\int_0^\infty \frac{2\cos\frac{\pi}{2}\xi}{1-\xi^2}\cos \xi x \, d\xi$$

$$1 = f(0) = \frac{2}{\pi}\int_0^\infty \frac{\cos\frac{\pi}{2}\xi}{1-\xi^2}\, d\xi$$