Chapter 2

Linear Time-Invariant Systems

(LTI Systems)
Linear Time-Invariant Systems

A system is said to be Linear Time-Invariant (LTI) if it possesses the basic system properties of linearity and time-invariance.

The input-output relationship for LTI systems is described in terms of a convolution operation.

**DT Signal Decomposition in terms of shifted unit impulses**

\[ x[n] = \sum_{k=-\infty}^{\infty} x[k] \delta[n-k] \]

Sum of scaled impulses

Impulse response

\[ h[n] \delta[n-k] \]

Time invariance

\[ h[n-k] \]

Linearity

\[ \sum \beta \delta[n-k] \]

\[ \sum \beta h[n-k] \]

Convolution sum

\[ y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k] \]

\[ y[n] = x[n] \ast h[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k] \]
Example:

Impulse response of an LTI system

There are only two non-zero values for the input

\[ y[n] = x[n] \ast h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n - k] \]

There are only two non-zero values for the input

\[ y[n] = x[0]h[n - 0] + x[1]h[n - 1] = 0.5h[n] + 2h[n - 1] \]
Example:

\[
x[n] = \begin{cases} 
  3 & n = 0 \\
  2 & n = 1 \\
  1 & n = 2 \\
  0 & \text{elsewhere}
\end{cases}
\]

\[
h[n] = \begin{cases} 
  3 & n = 0 \\
  1 & n = 1 \\
  2 & n = 2 \\
  0 & \text{elsewhere}
\end{cases}
\]

Solution:

Convolution sum using the table method.

<table>
<thead>
<tr>
<th>(k)</th>
<th>(-2)</th>
<th>(-1)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x(k))</td>
<td></td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(h(-k))</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(h(1-k))</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(h(2-k))</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(h(3-k))</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(h(4-k))</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(h(5-k))</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[y(0) = 3 \times 3 = 9\]
\[y(1) = 3 \times 2 + 1 \times 3 = 9\]
\[y(2) = 3 \times 1 + 1 \times 2 + 2 \times 3 = 11\]
\[y(3) = 1 \times 1 + 2 \times 2 = 5\]
\[y(4) = 2 \times 1 = 2\]
\[y(5) = 0 \text{ (no overlap)}\]

**Convolution Length** = \(N_1 + N_2 - 1 = 3 + 3 - 1 = 5\)
**Convolution**

**Example:** Find the output of an LTI system having a unit impulse response \( h[n] = u[n] \), for the input \( x[n] = \alpha^n u[n] \)

\[
x[k]h[n - k] = \begin{cases} 
\alpha^k & 0 \leq k \leq n \\
0 & \text{otherwise}
\end{cases}
\]

Thus, for \( n \geq 0 \),

\[
y[n] = \sum_{k=0}^{n} \alpha^k = \frac{1 - \alpha^{n+1}}{1 - \alpha}
\]

Thus, for all \( n \),

\[
y[n] = \left( \frac{1 - \alpha^{n+1}}{1 - \alpha} \right) u[n]
\]

\[
\alpha \frac{s}{1 - \alpha} = \frac{1}{1 - \alpha^2} \Rightarrow s = \frac{1}{1 - \alpha^2} = 1 + \alpha + \alpha^2 + \ldots + \alpha^n
\]

\[
s - \alpha s = s - \alpha^{n+1}
\]
The Representation of Continuous-Time Signals in Terms of Impulses

\[ x(t) \text{ is approximated in terms of pulses or staircase} \]

**Defining:**

\[
\delta_{\Delta}(t) = \begin{cases} 
1/\Delta & 0 \leq t \leq \Delta \\
0 & \text{otherwise}
\end{cases}
\]

\[
\Delta\delta_{\Delta}(t) = \begin{cases} 
1 & 0 \leq t \leq \Delta \\
0 & \text{otherwise}
\end{cases}
\]

\[ \Delta\delta_{\Delta}(t) \text{ has a unit amplitude} \]

\[ \hat{x}(t) \text{: the approximation of } x(t) \]

**At } t = 0 \]

\[ \hat{x}(0) = x(0)\Delta\delta_{\Delta}(t) = \begin{cases} 
x(0) & 0 \leq t \leq \Delta \\
0 & \text{otherwise}
\end{cases} \]

**At } t = \Delta \]

\[ \hat{x}(\Delta) = x(\Delta)\Delta\delta_{\Delta}(t - \Delta) = \begin{cases} 
x(\Delta) & \Delta \leq t \leq 2\Delta \\
0 & \text{otherwise}
\end{cases} \]

**In general \: At } t = k\Delta \]

\[ \hat{x}(k\Delta) = x(k\Delta)\Delta\delta_{\Delta}(t - k\Delta) = \begin{cases} 
x(k\Delta) & k\Delta \leq t \leq (k + 1)\Delta \\
0 & \text{otherwise}
\end{cases} \]

**The complete pulse/staircase approximation of } x(t) \text{ is the sum} \]

\[ \hat{x}(t) = \cdots + \hat{x}(-\Delta) + \hat{x}(0) + \hat{x}(\Delta) + \cdots \]

\[ \hat{x}(t) = \sum_{k=-\infty}^{\infty} x(k\Delta)\delta_{\Delta}(t - k\Delta) \]
The Representation of CT Signals in Terms of Impulses

If we let $\Delta$ approach 0 $\hat{x}(t)$ becomes closer and closer and equals $x(t)$ in the limit of 0

$$x(t) = \lim_{\Delta \to 0} \hat{x}(t) = \lim_{\Delta \to 0} \sum_{k=-\infty}^{\infty} x(k\Delta)\delta(t - k\Delta)\Delta$$

In the limiting case the sum approaches integral (area):

$$\Delta \to 0; \delta_{\Delta}(t) \to \delta(t)$$

$$\sum_{k=-\infty}^{\infty} \ldots \Delta \to \int_{\tau=-\infty}^{\infty} \ldots \, d\tau$$

Consequently:

$$x(t) = \int_{\tau=-\infty}^{\infty} x(\tau) \delta(t - \tau) \, d\tau$$
The Convolution Integral

- The impulse response \( h(t) \) of a continuous-time LTI system \( S \)
  \[ h(t) = S\{\delta(t)\} \]

  Sum (Integral) of weighted shifted impulses

For the input \( x(t) \):
\[
y(t) = S\{x(t)\} = S\left\{ \int_{\tau=-\infty}^{\infty} x(\tau) \delta(t-\tau) \, d\tau \right\} = \int_{\tau=-\infty}^{\infty} x(\tau) S\{\delta(t-\tau)\} \, d\tau
\]

- Since the system is time-invariant:
\[
S\{\delta(t-\tau)\} = h(t-\tau) \quad \text{Time-invariance}
\]

Example 1

Let, the input \( x(t) \) to an LTI system with unit impulse response \( h(t) \) be given as \( x(t) = e^{-at}u(t) \) for \( a > 0 \) and \( h(t) = u(t) \). Find the output \( y(t) \) of the system.

\[
y(t) = x(t) * h(t) = \int_{\tau=-\infty}^{\infty} x(\tau) h(t-\tau) \, d\tau
\]

\[
= \int_{0}^{\infty} e^{-a\tau} h(t-\tau) \, d\tau \quad \text{for } t > 0
\]

\[
= \int_{0}^{t} e^{-a\tau} \, d\tau = \frac{1}{-a} e^{-at} \bigg|_{0}^{t} = \frac{1}{a} (1 - e^{-at})
\]

Thus, for all \( t \), we can write
\[
y(t) = \frac{1}{a} (1 - e^{-at})
\]
Example 2

Find \( y(t) = x(t) \ast h(t) \), where

\[
\begin{align*}
x(t) &= e^{2t}u(-t) \\
h(t) &= u(t - 3)
\end{align*}
\]

The system response is \( y(t) = \int_{\tau=\infty}^{\tau=-\infty} x(\tau) h(t - \tau) \, d\tau \)

these two signals have regions of nonzero overlap

For \( t - 3 \leq 0 \): nonzero overlap for \( -\infty \leq \tau \leq t - 3 \)

\[
y(t) = \int_{-\infty}^{t-3} e^{2\tau} \, d\tau = \frac{1}{2} e^{2(t-3)} \quad \text{For } t \leq 3
\]

For \( t - 3 \geq 0 \): nonzero overlap for \( -\infty \leq \tau \leq 0 \)

\[
y(t) = \int_{-\infty}^{0} e^{2\tau} \, d\tau = \frac{1}{2} \quad \text{For } t \geq 3
\]

\[
y(t) = \begin{cases} 
\frac{1}{2} e^{2(t-3)} & \text{For } t \leq 3 \\
1/2 & \text{For } t \geq 3
\end{cases}
\]
The Convolution Integral

\[ y(t) = \int_{\tau = -\infty}^{\infty} x(\tau) h(t - \tau) \, d\tau \]

Convolution

Aggregate the overlapping

- \( t < 0 \) : No overlapping
- \( 0 < t < 2 \):
  \[ y(t) = \int_{0}^{t} 2 \cdot 1 \, d\tau \quad 0 < \tau < t \]
  \[ y(t) = 2 \left( t \bigg|_{0}^{t} \right) \]
  \[ y(t) = 2t \]

Shift \( h(t - \tau) \)

\( 0 < t < 2 \)
The Convolution Integral

\[ y(t) = \int_{-4}^{t} x(\tau)h(t-\tau) \, d\tau \]

- For \(2 < t < 4\):
  \[ y(t) = \int_{t-4}^{2} 2 \, d\tau = 2 \]
  \[ y(t) = 4 \]

- For \(4 < t < 6\):
  \[ y(t) = \begin{cases} 
  2 \cdot \tau \mid_{t-4}^{4} & \text{if } t-4 \leq \tau < 4 \\
  2(2-(t-4)) & \text{if } t-4 \leq \tau \leq 6 
  \end{cases} \]
  \[ y(t) = 2 \cdot (t-4) \]
  \[ y(t) = -2t + 12 \]

- For \(t > 6\):
  \[ y(t) = 0 \]

No-overlap
Properties of LTI Systems

- The characteristics of an LTI system are completely determined by its impulse response. This property holds in general only for LTI systems only.
- The unit impulse response of a nonlinear system does not completely characterize the behavior of the system.

Consider a discrete-time system with unit impulse response:

\[ h[n] = \begin{cases} 
1, & n = 0, 1 \\
0, & \text{otherwise}
\end{cases} \]

If the system is LTI, we get the system output (by convolution):

\[ y[n] = x[n] + x[n-1] \]

**There is only one such LTI system for the given \( h[n] \).**

However, there are many nonlinear systems with the same response, \( h[n] \).

- \( y[n] = (x[n] + x[n-1])^2 \)
- \( y[n] = \max(x[n], x[n-1]) \)

Two different Non-Linear systems with same impulse response:

\[ h[n] = (\delta[n] + \delta[n-1])^2 \]
\[ h[n] = \max(\delta[n], \delta[n-1]) \]

\[ h[n] = \begin{cases} 
1, & n = 0, 1 \\
0, & \text{otherwise}
\end{cases} \]
1 Commutative Property

\[ x(t) * h(t) = h(t) * x(t) \]
\[ x[n] * h[n] = h[n] * x[n] \]

Proof: (discrete domain)

\[ x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k] \]

Put \( r = n - k \Rightarrow k = n - r \)

\[ x[n] * h[n] = \sum_{r=-\infty}^{\infty} x[n-r] h[r] = \sum_{r=-\infty}^{\infty} h[r] x[n-r] = h[n] * x[n] \]

Similarly, we can prove it for continuous domain.
2 Distributive Property

Convolution is distributive over addition,

**in discrete time**
\[ x[n] \ast (h_1[n] + h_2[n]) = x[n] \ast h_1[n] + x[n] \ast h_2[n] \]

**in continuous time**
\[ x(t) \ast [h_1(t) + h_2(t)] = x(t) \ast h_1(t) + x(t) \ast h_2(t). \]

Example:

\[
y[n] = x[n] \ast h[n] = x[n] = \left(\frac{1}{2}\right)^n u[n] + 2^n u[-n]
\]

and
\[
h[n] = u[n]
\]

\[ x[n] \text{ in nonzero for entire } n, \text{ so direct convolution is difficult. Therefore, we will use commutative property.} \]

\[
y[n] = x[n] \ast h[n] = (x_1[n] + x_2[n]) \ast h[n] = (x_1[n] \ast h[n] + x_2[n] \ast h[n]) = y_1[n] + y_2[n]
\]

\[
y_1[n] = x_1[n] \ast h[n] = \sum_{k=-\infty}^{\infty} x_1[k] h[n-k] = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^k u[k] u[n-k] = \left(\frac{1}{2}\right)^{1}\left(1 - \frac{1}{2}\right)^{n+1} u[n] = 2 \left(1 - \left(\frac{1}{2}\right)^{n+1}\right) u[n]
\]

\[
y_2[n] = x_2[n] \ast h[n] = \sum_{k=-\infty}^{\infty} 2^k u[-k] u[n-k] = 2^{n+1}
\]
### 3 Associative Property

**In continuous time**

\[ x(t) * [h_1(t) * h_2(t)] = [x(t) * h_1(t)] * h_2(t) \]

**In discrete time**

\[ x[n] * [h_1[n] * h_2[n]] = [x[n] * h_1[n]] * h_2[n] \]

---

**Proof:**

From (A),

\[ y[n] = w[n] * h_2[n] = (x[n] * h_1[n]) * h_2[n] \]

From (B),

\[ y[n] = x[n] * h[n] = x[n] * (h_1[n] * h_2[n]) \]
A system is *memory-less* if its output at any time depends only on the value of the input at that same time.

System output:

\[ y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] \]

\[ \text{y}[n] \text{ depends on only } x[n] \text{ only if } k = n, \text{ so for } h[n] = 0 \text{ for } n \neq 0 \]

A discrete-time LTI system can be memory-less if only:

\[ h[n] = 0, \text{ for } n \neq 0 \]

impulse response \( x[n] = \delta[n] \)

\[ y[n] = x[n] \cdot h[0] = K\delta[n] \]

Thus, the impulse response have the form:

\[ h[n] = K\delta[n], \text{ with } K = h[0] \text{ is a constant} \]

the convolution sum reduces to

\[ y[n] = Kx[n] \]

If \( k = 1 \), then the system is called *identity system*. 

**Similarly for continuous LTI systems.**

A continuous-time LTI system is memory-less if

\[ h(t) = 0 \text{ for } t \neq 0, \]

\[ y(t) = Kx(t) \]
5 Invertibility of LTI Systems

A system is invertible only if *an inverse system exists*

The system with impulse response $h_1(t)$ is inverse of the system with impulse response $h(t)$ if:

$$h(t) * h_1(t) = \delta(t)$$

**Example:**

Consider the LTI system consisting of a pure time shift $y(t) = x(t - t_0)$ $t_0 > 0$ **delay**

The *impulse response* for the system (for $x(t) = \delta(t)$):

$$h(t) = \delta(t - t_0)$$

the system’s output (*the convolution*):

$$y(t) = x(t) * h(t) = x(t) * \delta(t - t_0) = x(t - t_0)$$

To recover the input from the output (*invert the system*), all that is required is to shift the output back.

*The inverse system impulse response:* $h_1(t) = \delta(t + t_0)$

then

$$h(t) * h_1(t) = \delta(t - t_0) * \delta(t + t_0) = \delta(t)$$

(identity system $y(t) = x(t) * \delta(t) = x(t)$)
Invertibility of LTI Systems: Example 2

Consider an LTI system with impulse response: \( h[n] = u[n] \)

Response of this system (convolution sum): \( y[n] = \sum_{k=-\infty}^{\infty} x[k] u[n - k] \)

\[
y[n] = \sum_{k=-\infty}^{\infty} x[k] u[n - k] = \sum_{k=-\infty}^{n} x[k]
\]

Impulse response (\( x[n] = \delta[n] \)): \( h_1[n] = \delta[n] - \delta[n - 1] \)

Verification: \( h[n] \ast h_1[n] = \delta[n] \)

\[
\begin{align*}
h[n] \ast h_1[n] & = u[n] \ast \{ \delta[n] - \delta[n - 1] \} \\
& = \{ u[n] \ast \delta[n] \} - \{ u[n] \ast \delta[n - 1] \} \\
& = u[n] - u[n - 1] \\
& = \delta[n]
\end{align*}
\]

the impulse responses are inverses of each other
6 Causality of LTI Systems

• The output of a *causal system* depends only on the present and past values of the input to the system.

\[ y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] \]

\( y[n] \) must not depend on \( x[k] \) for \( k > n \) to be *causal*

Therefore, for a discrete-time LTI system to be causal: \( x[k]h[n-k] = 0 \) for \( k > n \) \( \Rightarrow h[n-k] = 0 \) for \( k > n \)

\[ \text{for } k > n \rightarrow n-k < 0 \Rightarrow h[n] = 0 \quad \text{for } n < 0 \]

Causality for LTI system is equivalent to the condition of initial rest (output must be 0 before applying the input)

\[ \text{for } k > n \quad h[n-k] = 0 \quad \text{for } k < 0 \quad h[k] = 0 \]

Both the accumulator \((h[n] = u[n])\) and its inverse \((h[n] = \delta[n] - \delta[n-1])\) are causal.

Inverse system of the accumulator

\[ h[n]*h_1[n] = u[n]*\{\delta[n] - \delta[n-1]\} \]
\[ = u[n]*\delta[n] - u[n]*\delta[n-1] \]
\[ = u[n] - u[n-1] \]
\[ = \delta[n]. \]

• Similarly, for a continuous-time LTI system to be causal:

\[ y(t) = \int_{0}^{\infty} h(\tau)x(t-\tau)d\tau \]
7 Stability of LTI Systems

A system is stable if every bounded input produces a bounded output (BIBO).

Consider, an input $x[n]$ to an LTI system that is bounded in magnitude:

$$|x[n]| < B, \quad \text{for all } n$$

Suppose that we apply this to the LTI system with impulse response $h[n]$.

$$|y[n]| = \left| \sum_{k=-\infty}^{\infty} h[k] x[n-k] \right| \leq \sum_{k=-\infty}^{\infty} |h[k]| |x[n-k]| \leq B \sum_{k=-\infty}^{\infty} |h[k]| \quad \text{for all } n$$

We take $x[n] = B$

Therefore, if $\sum_{k=-\infty}^{\infty} |h[k]| < \infty$, then $|y[n]| < \infty$

The system is stable if the impulse response $h[n]$ is absolutely summable.

Similar case in continuous-time LTI system.

The system is stable if the impulse response is absolutely integrable.

Example:

An LTI system with pure time shift is stable.

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-\infty}^{\infty} |\delta[n-n_0]| = 1$$

An accumulator (DT domain) system is unstable.

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-\infty}^{\infty} |u[n]| = \sum_{n=0}^{\infty} |u[n]| = \infty$$

Similarly, an integrator (CT domain) system is unstable.
8 Unit Step Response of An LTI System

- The **unit step response**, \( s[n] \) or \( s(t) \), the output corresponding to the input \( x[n] = u[n] \) or \( x(t) = u(t) \).
- It is worthwhile relating the **unit step response** to the impulse response.

\[
s[n] = u[n] * h[n] = h[n] * u[n]
\]

\( u[n] \) is the **unit impulse response** of the **accumulator**.

\[
\Rightarrow s[n] = \sum_{k=-\infty}^{n} h[k]
\]

\[
\Rightarrow h[n] = s[n] - s[n-1]
\]

Response to the input \( h[n] \) of a LTI system with unit impulse response \( u[n] \).

\[
h[n] = \sum_{k=-\infty}^{n} \delta[k] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases} = u[n]
\]

- Running Sum
- First Difference
- Running Integral
- First Derivative

Discrete-time domain → Continuous-time domain
LTI Systems Described by Differential Equation
(Linear Constant-Coefficient Differential Equation)

A general N\textsuperscript{th}-order linear constant-coefficient differential equation that relates the input \(x(t)\) to the output \(y(t)\) is given by

\[
\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}
\]

Example 1:

Consider a first-order differential equation

\[
\frac{dy(t)}{dt} + 2y(t) = x(t)
\]

where the input to the system is:

\[
x(t) = K e^{3t} u(t)
\]

The complete solution is

\[
y(t) = y_p(t) + y_h(t)
\]

- Finding the particular solution \(y_p(t)\) (signal of the same form as the input)

  \[
y_p(t) = Y e^{3t}
\]

  \[
  3Y e^{3t} + 2Y e^{3t} = K e^{3t} \Rightarrow 3Y + 2Y = K \Rightarrow Y = \frac{K}{5}
  \]

  \[
y_p(t) = \frac{K}{5} e^{3t}, \text{ } K \text{ real and } t > 0
  \]

- Finding the homogeneous solution (hypothese a solution)

  \[
y_h(t) = A e^{st}
\]

  From differential equation:

  \[
sA e^{st} + 2A e^{st} = 0 \Rightarrow A(s + 2)e^{st} = 0 \Rightarrow s = -2
  \]

  \[
y_h(t) = A e^{-2t}
  \]

Complete solution:

\[
y(t) = \frac{K}{5} e^{3t} + A e^{-2t}
\]
Example_contd

To find $A$ suppose that the auxiliary condition is $y(0) = 0$, i.e., at $t = 0, y(t) = 0$

Using this condition into the complete solution, we get:

$$y(t) = \frac{K}{5} e^{3t} + Ae^{-2t} \text{ with } y(0) = 0$$

$$0 = \frac{K}{5} + A \Rightarrow A = -\frac{K}{5}$$

Example2:

Find $y[n]$ of the system with the difference equation $y[n] - \frac{1}{2}y[n-1] = x[n]$

We have the output $y[n] = x[n] + \frac{1}{2}y[n-1]$

Consider the input $x[n] = k \delta[n]$ and initial condition $y[-1] = 0$ (rest)

$$y[0] = x[0] + \frac{1}{2}y[-1] = k$$
$$y[1] = x[1] + \frac{1}{2}y[0] = \frac{1}{2}k$$
$$y[2] = x[2] + \frac{1}{2}y[1] = \left(\frac{1}{2}\right)^2 k$$

$$\vdots$$

$$y[n] = x[n] + \frac{1}{2}y[n-1] = \left(\frac{1}{2}\right)^n k$$

Setting $k = 1$ we obtain the impulse response for the system

$$h[n] = \left(\frac{1}{2}\right)^n u[n]$$

impulse response with infinite duration

infinite impulse response (IIR) systems.
Block Diagram Representations of Systems

Systems described by linear constant-coefficient difference and differential equations can be represented in terms of block diagram interconnections of elementary operations (adder, scaler, unit delay).

Example:

Consider the causal system described by the first-order difference equation

\[ y[n] + a y[n-1] = b x[n] \]

Consider the causal continuous-time system described by the first-order differential equation

\[ \frac{dy(t)}{dt} + ay(t) = b x(t) \]