PHYSICS 507
2nd HOMEWORK-SPRING 2020
Prof. V. Lempesis

Hand in: Monday 17 February at 23:59

1. In problem 3.17 we showed, in the class, that in general the superposition principle for electrostatic field energy does not hold.

A) Could you show under which condition the superposition principle may hold for the total energy of a field created by the superposition of two other fieds $\mathbf{E}_1$ and $\mathbf{E}_2$. (2 marks)

B) Could you suggest any such configuration made up by fields we have already seen in the class? (For example, spherical, ring, infinite wire, short wire, infinite plane etc) (1 mark)

Hint: Use the results of problem 3.17

Solution:

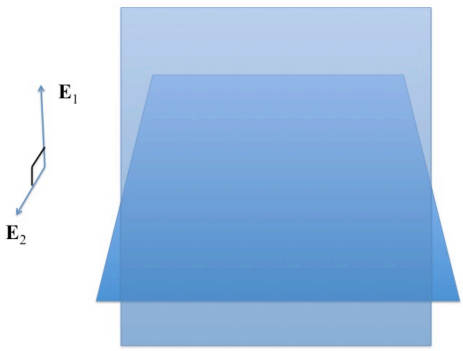
A) The result in Q.3.17 was:

$$W = W_1 + W_2 + \varepsilon_0 \int_{Vol} \mathbf{E}_1 \cdot \mathbf{E}_2 \, d\tau$$

So we may have superposition for the energies when $\mathbf{E}_1 \cdot \mathbf{E}_2 = 0$, or in other words, when we have two electric fields orthogonal to each other in the whole space.

B) Yes if we put two infinite charged sheets at right angles to each other we can create two electric fields perpendicular to each other. See the figure below for two positively charged infinite sheets.

Vasileios Lembessis 18/2/2020 17:13
Comment [1]: In order to have the superposition principle for energies valid, you need to find a field configurationwhere the two fields are perpendicular AT ANY POINT! If you find a confirguration where this occurs at only one or few points this is not correct. The reson is that the total energy is calculated after integration in all space. So the two fields have to be orthogonal EVERYWHERE!



2. The electric field at a point on the axis (take it z-axis) of a ring of radius *r*, and with uniform charge density *λ* (positive) is given by:

$$\mathbf{E} = \frac{\lambda}{2\varepsilon_0}\frac{zr}{\left(z^2+r^2\right)^{3/2}}\hat{\mathbf{k}}$$

A) Derive an expression for the electric potential at this point. (4 marks).

B) What is the value of the potential as $z >> r$? (1 mark)

Solution:

$$V = -\int_{\infty}^{z}\mathbf{E}\cdot d\mathbf{l} = -\int_{\infty}^{z}\frac{\lambda}{2\varepsilon_0}\frac{z'r}{\left(z'^2+r^2\right)^{3/2}}dz' = -\frac{\lambda r}{2\varepsilon_0}\int_{\infty}^{z}\frac{z'}{\left(z'^2+r^2\right)^{3/2}}dz'$$

$$V = -\frac{\lambda r}{4\varepsilon_0}\int_{\infty}^{z}\frac{1}{\left(z'^2+r^2\right)^{3/2}}dz'^2 \Rightarrow V = -\frac{\lambda r}{4\varepsilon_0}\int_{\infty}^{z}\frac{1}{\left(z'^2+r^2\right)^{3/2}}d\left(z'^2+r^2\right)$$

let $u^2 = z^2 + r^2$ then

$$V = -\frac{\lambda r}{4\varepsilon_0}\int_{\infty}^{z^2+r^2}\frac{1}{u^{3/2}}du \Rightarrow V = -\frac{\lambda r}{4\varepsilon_0}\frac{u^{-1/2}}{-1/2}\Bigg|_{\infty}^{z^2+r^2} \Rightarrow$$

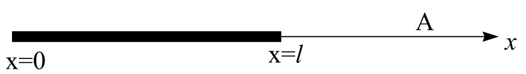
$$V = -\frac{\lambda r}{4\varepsilon_0}\int_{\infty}^{z^2+r^2}\frac{1}{u^{3/2}}du \Rightarrow V = \frac{\lambda r}{2\varepsilon_0}\left[\left(z^2+r^2\right)^{-1/2} - \infty^{-1/2}\right] \Rightarrow V = \frac{\lambda r}{2\varepsilon_0}\frac{1}{\sqrt{z^2+r^2}}$$

B) If $z >> r$ we get:

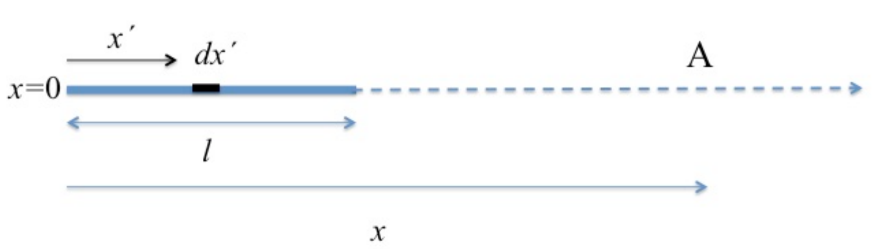
$$V = \frac{\lambda r}{2\varepsilon_0}\frac{1}{\sqrt{z^2+r^2}} \approx \frac{\lambda r}{2\varepsilon_0 z} = \frac{\lambda 2\pi r}{4\pi\varepsilon_0 z} = \frac{q}{4\pi\varepsilon_0 z}$$

As we would expect for a point like charge *q*. Where, here, *q* is the total charge of the ring.

3. A rod of length *l* is uniformly charged with a linear charge density λ. Find the electric potential at point A. (7 marks)



Solution



Assume an elementary length dx on the wire which carries an elementary charge $dq = \lambda dx'$. This elementary charge creates at point A an elementary potential given by:

$$dV_A = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\left(x-x'\right)}$$

So for the total potential at A we have after integrating on the charge distribution:

$$dV_A = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx'}{\left(x-x'\right)} \Rightarrow V_A = \frac{\lambda}{4\pi\varepsilon_0}\int_0^l \frac{dx'}{\left(x-x'\right)} \Rightarrow V_A = -\frac{\lambda}{4\pi\varepsilon_0}\int_0^l \frac{d\left(x-x'\right)}{\left(x-x'\right)}$$

$$V_A = -\frac{\lambda}{4\pi\varepsilon_0}\ln\left|x-x'\right|_0^l \Rightarrow V_A = -\frac{\lambda}{4\pi\varepsilon_0}\left\{\ln\left(x-l\right)-\ln x\right\}$$

$$V_A = \frac{\lambda}{4\pi\varepsilon_0}\left\{-\ln\left(x-l\right)+\ln x\right\} \Rightarrow V_A = \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{x}{x-l}\right)$$

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Comment [2]: Almost all of you solved the problem. OK. But you gave your result in terms of variables a, b etc. This is not a proper solution. Assume that you would have another question like: calculate the electric field from the expression of the potential. In such case you need to take the gradient of the potential etc. With your formalism you would have been lost. Conclusion: Express your answers in variables x, y, z (Cartesian) or ρ, φ, z (cylindrical) or r, θ, φ (Spherical). I just cut only one mark. Next time I will consider such answers as incomplete.

4. The potential of a static electric filed is given by $V = \sin(xy)$. What is the charge density of the field that creates this potential? (5 marks)

Solution:

$$\vec{\nabla}^2 V = -\rho/\varepsilon_0 \Rightarrow \rho = -\varepsilon_0\vec{\nabla}^2 V \Rightarrow \rho = -\varepsilon_0\left(\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}\right)$$

$$\rho = \varepsilon_0\left(\frac{\partial^2 \sin(xy)}{\partial x^2}+\frac{\partial^2 \sin(xy)}{\partial y^2}\right) \Rightarrow \rho = \varepsilon_0\left(x^2+y^2\right)\sin(xy)$$