(4.1)

1

$$(1-x^2)y'' - 2xy' + n(n+1)y = 0 \qquad n=3$$

نعوض مباشرة

$$P_3(x) = \frac{1}{2}(5x^3 - 3x) = y$$

$$P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3) = y$$

2

$$\{1, x, x^2, x^3, x^4, x^5\}, \quad -1 \le x \le 1$$

$$u_0 = P_0(x) \qquad \|u_0\|^2 = \cancel{\int_{-1}^{1} 1^2\,dx = 2}$$

$$v_0 = \frac{u_0}{\|u_0\|} \qquad \|1\| \qquad \int_{-1}^{1} x^2\,dx$$

[illegible]

29

$$v_0 = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}(1) = \frac{1}{\sqrt{2}} P_0(x) \quad \boxed{\sqrt{\tfrac{1}{2}}\; P_0(x)}$$

$$v_1 = \frac{u_1 - \langle u_1, v_0\rangle v_0}{\| u_1 - \langle u_1, v_0\rangle v_0 \|} \quad : \int_{-1}^{1} \frac{1}{\sqrt{2}} x \, dx = \frac{1}{\sqrt{2}} \frac{x^2}{2}\Big|_{-1}^{1} = 0$$

$$\|u_1\| = \tfrac{2}{3}$$

$$v_1 = \frac{u_1}{\|u_1\|} = x = P_1(x) \qquad v_1 = \frac{x}{\sqrt{\tfrac{2}{3}}} = \sqrt{\tfrac{3}{2}}\, x = \boxed{\sqrt{\tfrac{3}{2}}\; P_1(x)}$$

$$v_2 = \frac{u_2 - \langle u_2, v_0\rangle v_0 - \langle u_2, v_1\rangle v_1}{\| u_2 - \langle u_2, v_0\rangle v_0 - \langle u_2, v_1\rangle v_1 \|}$$

$$\langle u_2, v_0\rangle = \int_{-1}^{1} \frac{1}{\sqrt{2}} x^2 \, dx = \frac{x^3}{3}\Big|_{-1}^{1} = \frac{2}{3\sqrt{2}} = \frac{\sqrt{2}}{3}$$

$$\langle u_2, v_1\rangle = \int_{-1}^{1} x^2 \sqrt{\tfrac{3}{2}}\, x \, dx = 0$$

$$u_2 - \langle u_2, v_0\rangle v_0 = x^2 - \frac{\sqrt{2}}{3}\frac{1}{\sqrt{2}} = x^2 - \frac{1}{3}$$

$$\|x^2 - \tfrac{1}{3}\|^2 = \int_{-1}^{1} (x^2 - \tfrac{1}{3})^2 dx = \int_{-1}^{1} \left(x^4 - \tfrac{2}{3}x^2 + \tfrac{1}{9}\right) dx = \tfrac{1}{5}x^5 - \tfrac{2}{9}x^3 + \tfrac{1}{9}x \Big]_{-1}^{1}$$

$$= \frac{2}{5} - \frac{4}{9} + \frac{2}{9} = \frac{2}{5} - \frac{2}{9} = \frac{8}{45}$$

$$\|x^2 - \tfrac{1}{3}\| = \sqrt{\tfrac{8}{45}} = \frac{2\sqrt{2}}{3\sqrt{5}} = \frac{2}{3}\sqrt{\tfrac{2}{5}}$$

$$\frac{x^2 - \frac{1}{3}}{\frac{2}{3}\sqrt{\frac{2}{5}}} = \frac{3}{2}\sqrt{\tfrac{5}{2}}\left(x^2 - \tfrac{1}{3}\right) = \sqrt{\tfrac{5}{2}}\left(\frac{3x^2 - 1}{2}\right) = \boxed{\sqrt{\tfrac{5}{2}}\; P_2(x)}$$

(7) $x = \cos\theta \qquad \sin\theta \dfrac{d^2y}{d\theta^2} + \cos\theta \dfrac{dy}{d\theta} + n(n+1)\sin\theta\, y = 0$

(0.) ~~سؤال~~ ؟

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y=0 \qquad 0\le\theta\le\pi$$

$$\frac{dy}{dx}=\frac{dy}{d\theta}\frac{d\theta}{dx}=\frac{dy}{d\theta}\cdot\frac{1}{\frac{dx}{d\theta}}=-\csc\theta\frac{dy}{d\theta} \ast$$ ؛ أولاً

$$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{d\theta}\left(-\csc\theta\frac{dy}{d\theta}\right)\left(-\csc\theta\right) \quad \boxed{=\frac{1}{\frac{dx}{d\theta}}}$$

$$=-\csc^2\theta\left[\cot\theta\frac{dy}{d\theta}-\frac{d^2y}{d\theta^2}\right]$$

$\ast$ نعوض في

$$\sin^2\theta(-\csc^2\theta)\left[\cot\theta\frac{dy}{d\theta}-\frac{d^2y}{d\theta^2}\right]-$$

$$2\cos\theta\left[-\csc\theta\frac{dy}{d\theta}\right]+n(n+1)y=0$$

$$\frac{d^2y}{d\theta^2}-\frac{\cos\theta}{\sin\theta}\frac{dy}{d\theta}+2\frac{\cos\theta}{\sin\theta}\frac{dy}{d\theta}+n(n+1)y=0$$

$$\sin\theta\frac{d^2y}{d\theta^2}+\cos\theta\frac{dy}{d\theta}+n(n+1)\sin\theta\, y=0$$

(5)

$$(x^2-1)^n=\sum_{k=0}^{n}(-1)^k\frac{n!}{k!(n-k)!}x^{2n-2k}$$

$$P_n(x)=\sum_{k=0}^{[n/2]}(-1)^k\frac{(2n-2k)!}{2^n k!(n-k)!(n-2k)!}x^{n-2k}$$

؛ الحل

$$P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}\left[(x^2-1)^n\right]$$

$$\frac{1}{2^n n!}\frac{d^n}{dx^n}\left[(x^2-1)^n\right]=\frac{1}{2^n n!}\sum(-1)^k\frac{n!}{k!(n-k)!}$$

(01) [illegible]

$$\left((2n-2k)(2n-2k-1)(2n-2k-2)\cdots\left(\overset{(n-2k+1)}{2n-2k-(n-1)}\right)\right)X^{n-2k}$$

$$= \sum_{k=0}^{[n/2]} (-1)^k \frac{\overbrace{(2n-2k)(2n-2k-1)(2n-2k-2)\cdots(2n-2k-(n-1))}^{(2n-2k)!}X^{n}}{\text{المقام}}$$

$$= \sum_{k=0}^{[n/2]} (-1)^k \frac{(2n-2k)(2n-2k-1)\cdots(n-2k+1)(n-2k)!}{2^n\, k!\,(n-k)!\,(n-2k)!} X^{n-2k}$$

نضرب البسط والمقام ب $(n-2k)!$

$$= \sum_{k=0}^{[n/2]} (-1)^k \frac{(2n-2k)!}{2^n\, k!\,(n-k)!\,(n-2k)!} X^{n-2k}$$

وهو المطلوب (6) وهو ص