قسـم الرياضـيات	**316 ريـض**	**الفصـل الأول**
جامعة الملك سعود	**الاختبار النهائي**	**2012-2011**

1- حدد المعاملات a، b لكي تحقق الدالة $f(x)=ax+b$ المعادلتين

$$\langle f(x),x\rangle=1,\quad \|f(x)\|=2$$

في $\mathcal{L}^2(-1,1)$.

2- (أ) أوجد منشور فوريه للدالة $f(x)=x$ على الفترة $[-10,10]$.

(ب) هل التقارب منتظم؟ ولماذا؟

3- (أ) أثبت أن كثيرات حدود هرميت $H_n(x)$ متعامدة في الفضاء $\mathcal{L}^2\left(\mathbb{R},e^{-x^2}\right)$.

(ب) عبر عن x^3 بدلالة كثيرات هرميت.

4-.أثبت أن $J_{-n}(x)=(-1)^n J_n(x)$ لكل عدد طبيعي n.

5- أثبت أن

$$\frac{d}{dx}\left[x^{-\nu}J_\nu(x)\right]=-x^{-\nu}J_{\nu+1}(x)\qquad \forall\nu\geq 0.$$

6- (أ) أوجد تحويل فوريه بالصيغة المثلثية للدالة

$$f(x)=\begin{cases}\cos x, & |x|<\pi,\\ 0, & |x|>\pi,\end{cases}$$

ثم عبر عن f بتكامل فوريه.

(ب) استنتج من (أ) أن

$$\pi=2\int_0^\infty \frac{\xi}{1-\xi^2}\sin\pi\xi\,d\xi.$$

لا يكتب في
هذا الهامش

Q1: $f(x) = ax + b$

$\langle f(x), x \rangle = 1 \iff \int_{-1}^{1} x f(x) = 1$

$\Rightarrow \int_{-1}^{1} ax^2 + bx \, dx = 1$

$\Rightarrow \left[\frac{ax^3}{3} + \frac{bx^2}{2} \right]_{-1}^{1} = 1$

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$\left(\frac{a}{3} + \frac{b}{2} \right) - \left(\frac{-a}{3} + \frac{b}{2} \right) = 1$

$\frac{a}{3} + \cancel{\frac{b}{2}} + \frac{a}{3} - \cancel{\frac{b}{2}} = 1$

$\frac{2a}{3} = 1 \implies \boxed{a = \frac{3}{2}}$ ✓

$\| f(x) \| = 2 \iff \sqrt{\int_{-1}^{1} (ax+b)^2 dx} = 2$

$\int_{-1}^{1} a^2x^2 + 2abx + b^2 \, dx$

$= \left[\frac{a^2x^3}{3} + abx^2 + b^2x \right]_{-1}^{1}$

لا يكتب في
هذا الهامش

$$f(x) = \sum b_n \sin \frac{n\pi x}{L}$$

$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin \frac{n\pi x}{L}$$

$$= \frac{1}{10} \int_{-10}^{10} x \sin \frac{n\pi x}{10} \, dx \quad \checkmark$$

odd x odd = even

$$= \frac{2}{10} \int_{0}^{10} x \sin \frac{n\pi x}{10} \, dx \qquad M = \frac{n\pi}{10}$$

$$= \frac{1}{5} \int_{0}^{10} x \sin Mx \, dx$$

بالجزء

$u = x \qquad dv = \sin Mx$

$du = 1 \qquad v = \dfrac{-\cos Mx}{M}$

$$\frac{1}{5} \left[\frac{-x \cos Mx}{M} \Big|_0^{10} + \int + \frac{\cos Mx}{M} \right]_0$$

$$\frac{1}{5} \left[\frac{-x \cos Mx}{M} \Big|_0^{10} + \frac{1}{M^2} \left[\sin Mx \right]_0^{10} \right]$$

$$\frac{1}{5} \left[-x \cos \frac{n\pi}{} x \Big|^{10} + \frac{1}{[illegible]} \left[\sin \frac{n\pi}{} x \right]^{10} \right]$$

$$f(x) = \sum b_n \sin \frac{n\pi x}{L}$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x) \sin \frac{n\pi x}{L}$$

$$= \frac{1}{10}\int_{-10}^{10} x \sin \frac{n\pi x}{10}\, dx \quad \checkmark$$

odd x odd = even

$$= \frac{2}{10}\int_{0}^{10} x \sin \frac{n\pi x}{10}\, dx \qquad \mu = \frac{n\pi}{10}$$

$$= \frac{1}{5}\int_{0}^{10} x \sin \mu x\, dx$$

بالتجزيء

$u = x \qquad dv = \sin \mu x$

$du = 1 \qquad v = \frac{-\cos \mu x}{\mu}$

$$\frac{1}{5}\left[-\frac{x \cos \mu x}{\mu}\Big|_0^{10} + \int + \frac{\cos \mu x}{\mu} \right]_0$$

$$\frac{1}{5}\left[-\frac{x \cos \mu x}{\mu}\Big|_0^{10} + \frac{1}{\mu^2}\left[\sin \mu x\right]_0^{10} \right]$$

$$\frac{1}{5}\left[-\frac{x \cos \frac{n\pi}{10} x}{\frac{n\pi}{10}}\Bigg|_0^{10} + \frac{1}{\cancel{\mu^2}\ \left(\frac{n\pi}{10}\right)^2}\left[\sin \frac{n\pi}{10} x\right]_0^{10} \right]$$

$$\frac{1}{5}\left[\left(\frac{-10\cos n\pi}{\frac{n\pi}{10}}+0\right)+\frac{10^2}{n^2\pi^2}\left[\sin n\pi-(0)\right]\right]$$

$$\frac{1}{5}\left[\frac{-100\cos n\pi}{n\pi}+\frac{100\cancel{\sin n\pi}}{n^2\pi^2}\right]$$

$$\frac{1}{5}\left[\frac{-100\cancel{n\pi}}{\cancel{n^2}\pi^2}\right.$$

$\sin n\pi = 0$

$\cos n\pi = (-1)^n$

$$\frac{1}{5}\left[\frac{-100(-1)^n}{n\pi}\right]=\frac{-20(-1)^n}{n\pi}$$

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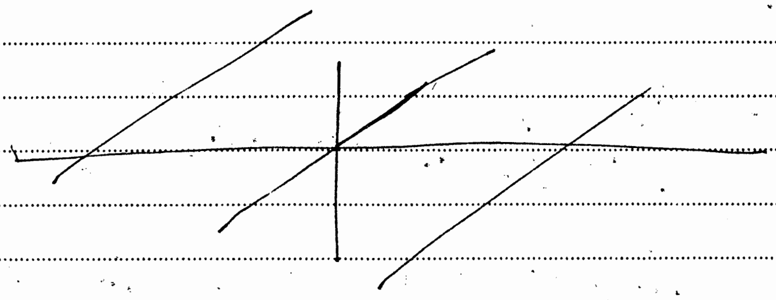
$$f(x)\,b_n=\sum\frac{-20(-1)^n}{n\pi}\sin\left(\frac{n\pi}{10}x\right)$$

~~$f(x)=x$ هي دالة متصلة على~~

~~IR لان كثيرة حدود ← السلسلة متقاربة~~

متصلة عن $[-10,10]\subset IR$ ~~لا بانتظام~~



لا يكتب في هذا الهامش

Q3 ب

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$\cancel{C_0}, C_1, \cancel{C_2}, C_3$

$f(x) = x^3$

$f(-x) = -f(x) \Longrightarrow$ odd is $f(x)$

$C_n = 0$ n even

$f(x) = x^3 = 0 H_0 + C_1 H_1 + 0 H_2 + C_3 H_3$

$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$

$H_0(x) = 1 \cancel{(0) = 0}$

$H_1(x) = (-1) e^{x^2} (-2x e^{-x^2})$

$= (-1)(-2x) = 2x$

$H_3(x) = (-1)^3 e^{x^2} ($

$y = e^{-x^2}$

$y' = -2x e^{-x^2}$

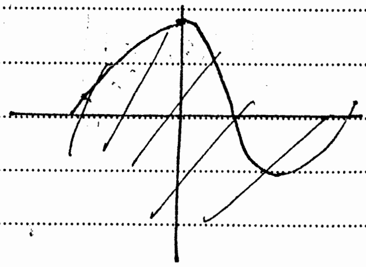
$y'' = -2[e^{-x^2} + x(-2x) e^{-x^2}]$

$= -2e^{-x^2}[1 - 2x^2]$

Q 6:

$-\pi < x < \pi$

$$f(x) = \begin{cases} \cos x & |x| < \pi \\ 0 & |x| > \pi \\ & x > \pi \text{ or } x < -\pi \end{cases}$$



$$f(x) = \frac{1}{\pi}\int_0^{\infty} A(\alpha)\cos\alpha x + B(\alpha)\sin\alpha x$$

$B(\alpha) = 0$ زوجية f

$$A(\alpha) = \int_{-\infty}^{\infty} f(x)\cos\alpha x \, dx$$

$$\int_{-\pi}^{\pi} \cos x \cos\alpha x \, dx = \cancel{2}\int_0^{\pi} \frac{\cos(1+\alpha)x + \cos(1-\alpha)x}{\cancel{2}} \, dx$$

زوجية

$$= \left[\frac{\sin(1+\alpha)x}{1+\alpha} + \frac{\sin(1-\alpha)x}{1-\alpha} \right]_0^{\pi}$$

$$= \left[\left(\frac{\sin(\pi+\alpha\pi)}{1+\alpha} + \frac{\sin(\pi-\alpha\pi)}{1-\alpha} \right) - (0+0) \right]$$

$$\frac{(1-\alpha)\left[\sin\pi\cos\alpha\pi + \cos\pi\sin\alpha\pi\right] + (1+\alpha)\left[\sin\pi\cos\alpha\pi - \cos\pi\sin\alpha\pi\right]}{1-\alpha^2}$$

$$\frac{(1-\alpha)\left[-\sin\alpha\pi\right] + (1+\alpha)\left[\sin\alpha\pi\right]}{1-\alpha^2} = \frac{\sin\alpha\pi\left[\alpha-1+1+\alpha\right]}{1-\alpha^2}$$

لا يكتب في
هذا الهامش

$$f(x) = \frac{1}{\pi}\int_0^{\infty} \frac{2\alpha \sin\alpha\pi}{1-\alpha^2} \cos\alpha x \, d\alpha$$

at $x = \pi$

$$1 = \frac{1}{\pi}\int_0^{\infty} \frac{2\alpha \sin\alpha\pi}{1-\alpha^2} \, d\alpha$$

$$1 = \frac{2}{\pi}\int_0^{\infty} \frac{2\alpha \sin\alpha\pi}{1-\alpha^2} \, d\alpha$$

$$\pi = 2\int_0^{\infty} \frac{2\alpha \sin\alpha\pi}{1-\alpha^2} \, d\alpha$$

لا يكتب في
هذا الهامش

$$(-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$$

$$\frac{x^{\nu}}{2^{\nu}} \sum \frac{x^{2m} (-1)^m}{2^{2m}\, m!\, \Gamma(m+\nu+1)}$$

$$\frac{x^{-\nu}}{2^{-\nu}} \sum \frac{x^{2m} (-1)^m}{\quad}$$

لا يكتب في
هذا الهامش

$(-1)e^{x^2}$

e^{-x^2} $2x$

$-2xe^{-x^2}$

$-2[e^{-x^2}+x(-2xe^{-x^2})]$ y''

$-2[-2e^{-x^2}[1-2x^2]$

$-2[-2xe^{-x^2}[1-2x^2]+e^{-x^2}(-4x)]$

$4x\cancel{e^{-x^2}}[1-2x^2]+4x\cancel{e^{-x^2}}$

$\cancel{4x-8x^3-4}$

$-2[-2x+4x^3+(-4x)]$

$-2[-6x+4x^3]$

$12x-8x^3$

$8x^3-12x$

$$-2\left[xe^{-x^2}\right]$$

y''

$$-2\left[e^{-x^2}+x(-2xe^{-x^2})\right]$$

$$-2e^{-x^2}\left[1 - 2x^2\right]$$

$$-2\left[-2xe^{-x^2}(1-2x^2)+e^{-x^2}(-4x)\right]$$

$$-2e^{-x^2}\left[-2x+4x^3-4x\right]$$

$$\cancel{-}2\cancel{e^{-x^2}}\left[-6x+4x^3\right]$$

$$8x^3-12x$$