

1. Determine the coefficients a_i and b_i in the expression

$$a_0 + a_1 \cos x + b_1 \sin x + a_2 \cos 2x + b_2 \sin 2x$$

which give the best approximation in $L^2(-\pi, \pi)$ of the function $f(x) = x$, $-\pi < x < \pi$.

2. (i) Let $L = a \frac{d^2}{dx^2} + b \frac{d}{dx} + c$ be a linear differential operator with constant coefficients. When is L formally self-adjoint?

(ii) Determine the eigenvalues and eigenfunctions of the operator $L = \frac{d^2}{dx^2} + 1$ on the interval $(0, 1)$ by solving the equation $Lu + \lambda u = 0$ under the boundary conditions

$$u(0) = 1, \quad \text{(ii) } u(1) = 0.$$

3. Determine the eigenvalues and eigenfunctions of the equation $x^2 u'' + xu' + \lambda u = 0$ on the interval $(1, e)$, subject to the conditions

$$u(1) = u(e) = 0.$$

Write the orthogonality relation between the eigenfunctions.

4. expand the function

$$f(x) = \begin{cases} 0, & -\pi < x < 0, \\ x, & 0 < x < \pi. \end{cases}$$

in a Fourier series. Is the convergence uniform, and why?