Thermal & Statistical Physics

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PHYS 343

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Lecture 6

Kinetic Theory of Gases

Ideal Gas Model

Ideal Gas Model

Ideal Gas Model

Ideal Gas Law

The equality for the four variables involved in Boyle's Law, Charles' Law, Gay-Lussac's Law and Avogadro's law can be written

$$PV = nRT$$

This is the equation of state for ideal gases

Ideal Gases

All gases approach a unique "ideal gas" at low densities.

An ideal gas obeys the **"ideal gas law"**

$$p V = n R T = N k T$$

Ludwig Boltzmann
1866- 1906

p = absolute pressure (Pa)

V = volume (m3)

n = number of moles

T = temperature (kelvin)

R = gas constant = 8.31 J/(mole K) = $k N_A$

k = Boltzmann's constant = $1.38\ 10^{-23}$ J/K

N = number of molecules

Boltzmann's constant

$$PV = nRT = nN_A\left(\frac{R}{N_A}\right)T = N\left(\frac{R}{N_A}\right)T$$

The constant term R/N_A is referred to as ***Boltzmann's constant,*** in honor of the Austrian physicist Ludwig Boltzmann (1844–1906), and is represented by the **symbol k:**

$$k = \frac{R}{N_A} = \frac{8.31\ \text{J/(mol}\cdot\text{K)}}{6.022\times10^{23}\ \text{mol}^{-1}} = 1.38\times10^{-23}\ \text{J/K}$$

PV = NkT

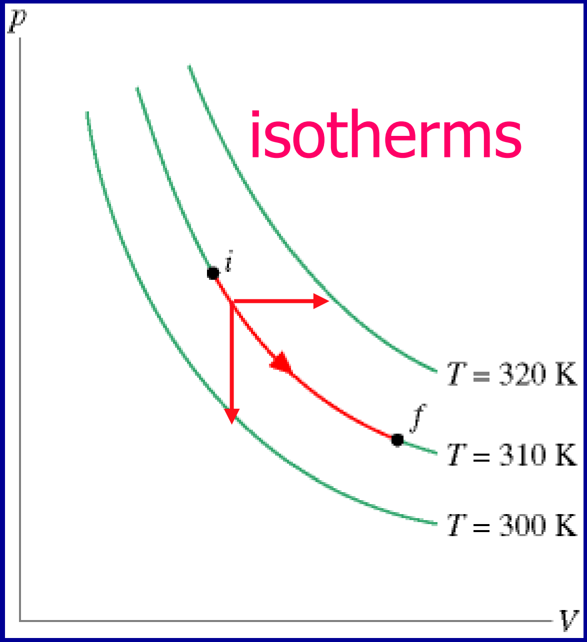
Work done by an ideal gas

$$pV = nRT \qquad W = \int_{V_i}^{V_f} p\,dV$$

- **Constant temperature:**

keep temperature constant, change the volume. The pressure will change, following

$$p = nRT/V$$



How much work is done by the gas?

$$W = nRT \ln (V_f/V_i)$$

- **Constant pressure:**

keep pressure constant, change the volume.
The temperature will change, following T=pV/nR.
How much work is done by the gas?

$$W = p\,\Delta V = p(V_f - V_i)$$

- **Constant volume:**

pressure and temperature may change, but no work is done!

$$W = 0$$

Learning Check

What is the value of R when the STP value for P is 760 mmHg?

Solution

What is the value of R when the STP value for P is 760 mmHg?

$$R = \frac{PV}{nT} = \frac{(760\ \text{mm Hg})\ (22.4\ \text{L})}{(1\text{mol})\quad(273\text{K})}$$

$$= 62.4\ \text{L-mm Hg/ mol-K}$$

Learning Check

Dinitrogen monoxide (N_2O), laughing gas, is used by dentists as an anesthetic. If 2.86 mol of gas occupies a 20.0 L tank at 23°C, what is the pressure (mmHg) in the tank in the dentist office?

Solution

Set up data for 3 of the 4 gas variables

Adjust to match the units of R

V =	20.0 L	20.0 L
T =	23°C + 273	296 K
n =	2.86 mol	2.86 mol
P =	?	?

Rearrange ideal gas law for unknown P

$$P = \frac{nRT}{V}$$

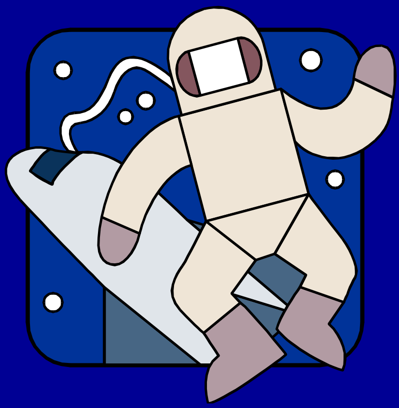
Substitute values of n, R, T and V and solve for P

$$P = \frac{(2.86\ \text{mol})(62.4\ \text{L-mmHg})(296\ \text{K})}{(20.0\ \text{L})\qquad (\text{K-mol})}$$

$$= 2.64 \times 10^3\ \text{mm Hg}$$

Learning Check

A 5.0 L cylinder contains oxygen gas at 20.0°C and 735 mm Hg. How many grams of oxygen are in the cylinder?



Solution

Solve ideal gas equation for n (moles)

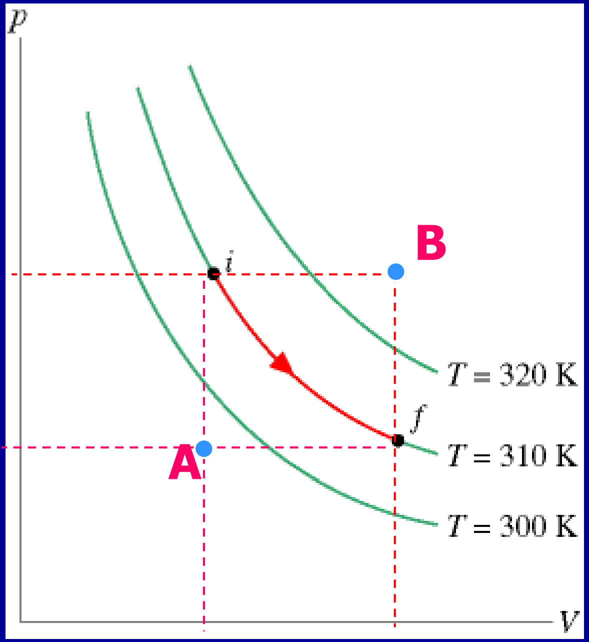
$$n = \frac{PV}{RT}$$

$$= \frac{(735\ \text{mmHg})(5.0\ \text{L})(\text{mol K})}{(62.4\ \text{mmHg L})(293\ \text{K})}$$

$$= 0.\ 20\ \text{mol } O_2 \times \frac{32.0\ \text{g } O_2}{1\ \text{mol } O_2} = 6.4\ \text{g } O_2$$

Example

A gas can be taken from the initial state *i* to the final state *f* in many different ways, usually following constant pressure curves, constant volume curves, and isotherms.



a) If the initial pressure is 1Pa, and the initial volume is $1m^3$, how many moles are there in the gas?
b) If the final volume is 1.1 m^3, what is the final pressure?
c) What is the path from i to f where the gas does minimum work?
d) What is the temperature at intermediate points A, B?
e) If the system is taken to the final state through the 310 K isotherm, and then back to the original state through point B, what is the total heat added to the system?

Some ideas…..

Types of Problems

$$\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2}$$

Given initial conditions, determine final conditions; Cancel out what is constant

- Make Substitution into

  PV = nRT

$$moles(n) = \frac{mass, g}{MolarMass, g / mole}$$

$$Density = \frac{mass}{Volume}$$

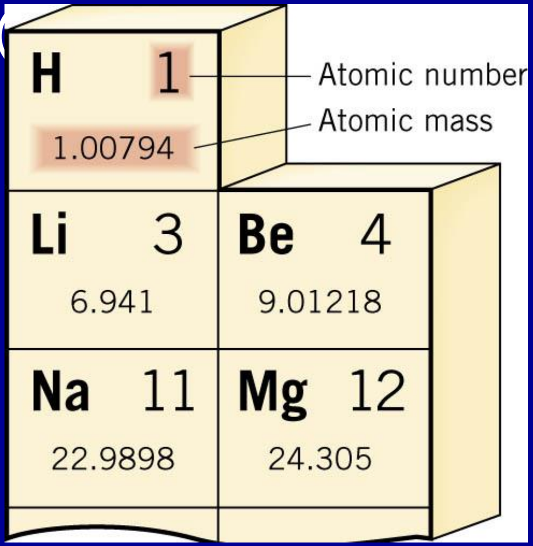
S.I ….
Molecular Mass

To facilitate comparison of the mass of one atom with another, a mass scale know as the ***atomic mass scale*** has been established.

$$1\,\mathrm{u} = 1.6605 \times 10^{-27}\,\mathrm{kg}$$

The unit is called the ***atomic mass unit*** (

The atomic mass is given in atomic mass units. For example, a Li atom has a mass of 6.941u.



S.I ....
The Mole

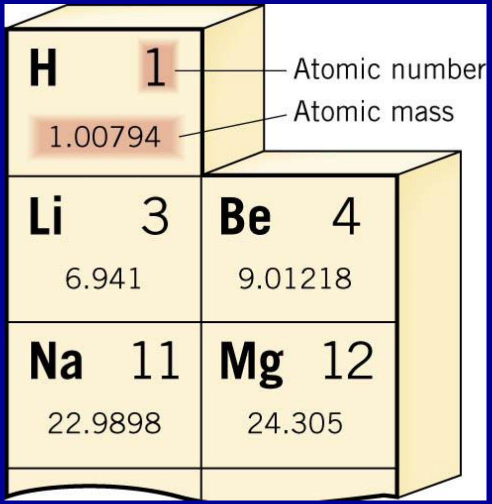
- The amount of gas in a given volume is conveniently expressed
- in terms of the number of moles
- One **mole** of any substance is that amount of the substance that contains **Avogadro's number** of constituent particles
  - Avogadro's number $N_A = 6.022 \times 10^{23}$
  - The constituent particles can be atoms or molecules

S.I ….
Moles, cont

- The number of moles can be determined from the mass of the substance:
- $$n = m / M$$
  - *M* is the molar mass of the substance
    - Can be obtained from the periodic table
    - Is the atomic mass expressed in grams/mole
      - Example: He has mass of 4.00 u so M = 4.00 g/mol
  - *m* is the mass of the sample
  - *n* is the number of moles

S.I ....
the Mole, and Avogadro's Number

One ***mole*** of a substance contains as many particles as there are atoms in 12 grams of the isotope cabron-12.



$$N_A = 6.022 \times 10^{23}\,\mathrm{mol}^{-1}$$

The number of moles can also be determined from the number of atoms of the substance:

$$n = \frac{N}{N_A}$$

number of moles

number of atoms

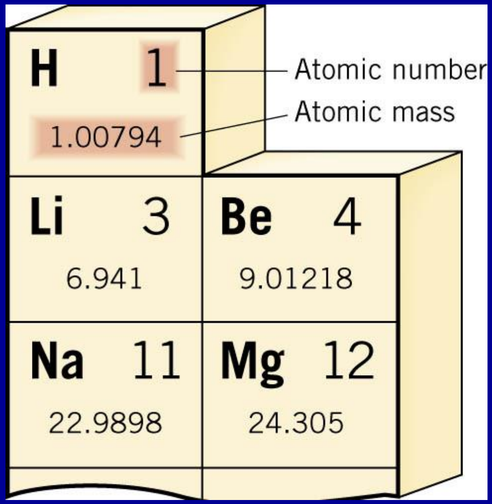
S.I ….

the Mole, and Avogadro's Number

$$n = \frac{m_{\text{particle}} N}{m_{\text{particle}} N_A} = \frac{m}{\text{Mass per mole}}$$

The mass per mole (in g/mol) of a substance has the same numerical value as the atomic or molecular mass of the substance (in u).

For example Hydrogen has an atomic mass of 1.00794 g/mol, while the mass of a single hydrogen atom is 1.00794 u.



S.I ....

Avogadro's number

- **An Avogadro's number** of standard soft drink cans would cover the surface of the earth to a depth of over 200 miles.
- If you had Avogadro's number of unpopped popcorn kernels, and spread them across the **Arabic world** , it would be covered in popcorn to a depth of over 9 miles.
- If we were able to count atoms at the rate of 10 million per second, it would take about 2 billion years to count the atoms in one mole.

. S.I …

the Mole, and Avogadro's Number

Example : **The Hope Diamond and the Rosser Reeves Ruby**

The Hope diamond (44.5 carats) is almost pure carbon. The Rosser Reeves ruby (138 carats) is primarily aluminum oxide (Al_2O_3). One carat is equivalent to a mass of 0.200 g. Determine (a) the number of carbon atoms in the Hope diamond and (b) the number of Al_2O_3 molecules in the ruby.

Solution:

(a)

$$n = \frac{m}{\text{Mass per mole}} = \frac{(44.5\ \text{carats})[(0.200\ \text{g})/(1\ \text{carat})]}{12.011\ \text{g/mol}} = 0.741\ \text{mol}$$

$$N = nN_A = (0.741\ \text{mol})(6.022\times10^{23}\ \text{mol}^{-1}) = 4.46\times10^{23}\ \text{atoms}$$

(b)

$$n = \frac{m}{\text{Mass per mole}} = \frac{(138\ \text{carats})[(0.200\ \text{g})/(1\ \text{carat})]}{\underbrace{101.96}_{2(26.98)+3(15.99)}\ \text{g/mol}} = 0.271\ \text{mol}$$

$$N = nN_A = (0.271\ \text{mol})(6.022\times10^{23}\ \text{mol}^{-1}) = 1.63\times10^{23}\ \text{atoms}$$

S.I ...

Converting Units of Pressure

Problem: A physicist collects a sample of carbon dioxide from the decomposition of limestone ($CaCO_3$) in a closed end manometer, the height of the mercury is 341.6 mm Hg. Calculate the CO_2 pressure in torr, atmospheres, and kilopascals.

Plan: The pressure is in mmHg, so we use the conversion factors from Table 5.2(p.178) to find the pressure in the other units.

Solution:

converting from mmHg to torr:

$$P_{CO2}\,(\text{torr}) = 341.6\ \text{mm Hg} \times \frac{1\ \text{torr}}{1\ \text{mm Hg}} = \mathbf{341.6\ torr}$$

converting from torr to atm:

$$P_{CO2}(\ \text{atm}) = 341.6\ \text{torr} \times \frac{1\ \text{atm}}{760\ \text{torr}} = \mathbf{0.4495\ atm}$$

converting from atm to kPa:

$$P_{CO2}(\text{kPa}) = 0.4495\ \text{atm} \times \frac{101.325\ \text{kPa}}{1\ \text{atm}} = \mathbf{45.54\ kPa}$$

S.I …

Molar Mass of a gas

What is the molar mass of a gas if 0.250 g of the gas occupy 215 mL at 0.813 atm and 30.0°C?

$$n = \frac{PV}{RT} = \frac{(0.813 \text{ atm})(0.215 \text{ L})}{(0.0821 \text{ L-atm/molK})(303\text{K})} = 0.00703 \text{ mol}$$

$$\text{Molar mass} = \frac{\# \text{ g}}{\# \text{ mol}} = \frac{0.250 \text{ g}}{0.00703 \text{ mol}} = 35.6 \text{ g/mol}$$

S.I …

Density of a Gas

Calculate the density in g/L of O_2 gas at STP.

From STP, we know the P and T.

P = 1.00 atm T = 273 K

Rearrange the ideal gas equation for moles/L

$$PV = nRT \qquad \frac{PV}{RTV} = \frac{nRT}{RTV} \qquad \frac{P}{RT} = \frac{n}{V}$$

Substitute

$$\frac{(1.00 \text{ atm}) \text{ mol-K}}{(0.0821 \text{ L-atm})(273 \text{ K})} = 0.0446 \text{ mol } O_2/\text{L}$$

Change moles/L to g/L

$$\frac{0.0446 \text{ mol } O_2}{1 \text{ L}} \times \frac{32.0 \text{ g } O_2}{1 \text{ mol } O_2} = 1.43 \text{ g/L}$$

Therefore the density of O_2 gas at STP is 1.43 grams per liter