DISCRETE PROBABILITY DISTRIBUTIONS

Bernoulli distribution

Q17) Suppose our class passed (C or better) the last exam with probability 0.75.

a. Find the probability that someone passes the exam.
b. Find the mean value of the random variable
c. Find the standard deviation value of the random variable
d. Find the moment generating function of the random variable (later)

Solution :

X:# of student pass the last exam. X is a Bernoulli r.v with $P = 0.75 \quad , n = 1$

$f_X(x) = \binom{1}{x}(0.75)^x(0.25)^{1-x} = (0.75)^x(0.25)^{1-x} \quad ; x = 0,1$

The r.v X has Bernoulli (0.75) where the success (X=1) is passing the exam, then

$P(X = 1) = 0.75$

$E(X) = p = 0.75$

$\sqrt{Var(x)} = \sigma = \sqrt{pq} = \sqrt{0.75 * 0.25} = 0.4330$

$M_X(t) = p\, e^t + q = 0.75\, e^t + 0.25$

Q18) 20% from a population have a particular disease. In testing process for infection by this disease.

a. Find the probability that someone infected by this disease.
b. Find the mean value of the random variable.
c. Find the standard deviation value of the random variable.
d. Find the moment generating function of the random variable. (later)

Solution :

X:# of person have disease. ; $P = 0.2 \quad , n = 1 ; X \sim Bernoulli\ (0.2)$

$f_X(x) = (0.2)^x(0.8)^{1-x} \quad ; x = 0,1$

a) $f_X(1) = 0.2$

b) $\mu = p = 0.2$

c) $\sigma = \sqrt{pq} = \sqrt{0.2 * 0.8} = \sqrt{0.16} = 0.4$

d) $M_X(t) = p\, e^t + q = (0.8) + (0.2)e^t$

Binomial distribution

Q1) In a certain city district the need for money to buy drugs is stated as the: reason for 75% of all thefts. Find the probability that among the next **5** theft cases reported in this district,

a. Exactly 2 resulted from the need for money to buy drugs.
b. At most 3 resulted from the need for money to buy drugs.

Solution :

X: # of thefts resulted from the need for money to buy drugs among the next 5 theft cases.

$$X \sim Binomial(5, 0.75) \quad \leftrightarrow \quad f(x) = \binom{5}{x} (0.75)^x (0.25)^{(5-x)} \ ; \ x = 0,1,2, \dots ,5$$

For n=5 and p=3/4 , we have

a) $P(X = 2) = \binom{5}{2} (0.75)^2 \ (0.25)^3 = 0.0879$

b) $P(X \leq 3) = \ 1 - P(X > 3) = 1 - P(\ X = 4) - \ P(X = 5)$

$$= 1 - \binom{5}{4} (0.75)^4 \ (0.25)^1 - \binom{5}{5} (0.75)^5 (0.25)^0 = 0.3672$$

Q2) In testing a certain kind of truck tire over a rugged terrain, it is found that 25% of the trucks fail to complete the test run without a blowout. Of the next 15 trucks tested, find the probability that

a. From 3 to 6 have blowouts.
b. Fewer than 4 have blowouts.
c. More than 5 have blowouts.

Solution :

X: no. of trucks having blowouts during the test run out of next 15 trucks tested.

$$X \sim Binomial(15, 0.25) \quad \leftrightarrow \quad f_X(x) = \binom{15}{x} \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{15-x} \quad , x = 0,1, \dots ,15$$

(a) $P(3 \leq X \leq 6) \ = \ P \ (X \leq 6) - P(X \leq 2) \ = 0.9434 - 0.2361 = 0.7073$

Or $P(3 \leq X \leq 6) \ = \sum_{x=3}^{6} f_X(x)$

(b) $P(X < 4) = P(X \leq 3) = \ 0.4613.$

Or $P(X < 4) = 1 - P(X \geq 4) = 1 - \sum_{x=4}^{15} f_X(x)$

(c) $P(X > 5) = 1 - P(X \leq 5) = 1 - 0.8516 = 0.1484$

Or $P(X > 6) = \sum_{x=6}^{15} f_X(x)$

Q3) The probability that a patient recovers from a delicate heart operation is 0.9. What is the probability that exactly 5 of the next 7 patients having this operation survive?

Solution :

X: # of patient that recovers from heart operation out of 7 patients.

$$X \sim Binomial(7\,, 0.9) \quad \leftrightarrow \quad f_X(x) = \binom{7}{x}(0.9)^x(0.1)^{7-x} \quad , x = 0,1, \ldots, 7$$

$$P(X = 5) = \binom{7}{5}(0.9)^5(0.1)^{7-5}$$

Q4) It is known that 60% of mice inoculated with a serum are protected from a certain disease. If 5 mice are inoculated, find the probability that

a. none contracts the disease.
b. fewer than 2 contract the disease.
c. more than 3 contract the disease.

Solution :

X: no. of mice that contract the disease out of 5 mice inoculated .
If a mouse contracts the disease, we consider that as a **success**.
If a mouse does not contract the disease, will be a **failure**.
Trials are independent.

$$X \sim Binomial(5\,, 0.4) \quad \leftrightarrow \quad f_X(x) = \binom{5}{x}(0.4)^x(0.6)^{5-x} \quad , x = 0,1, \ldots, 5$$

(a) P(X=0)= 0.0778

(b) $P(X < 2) = P(X \le 1) = \sum_{x=0}^{1} f(x) = 0.3370$

(c) $P(X > 3) = \sum_{x=4}^{5} f(x) = 0.0870$

Or $= 1 - P(X \le 3) = 1 - \sum_{x=0}^{3} f(x) = 1 - 0.9130 = 0.0870$

Q5) In a study of brand recognition, 95% of consumers recognized Coke. The company randomly selects 4 consumers for a taste test. Let *X* be the number of consumers who recognize (of 4 consumers).

a. Write out the PMF table for this.
b. Find the probability that among the 4 consumers, 2 or more will recognize Coke.
c. Find the expected number of consumers who will recognize Coke.
d. Find the variance for the number of consumers who will recognize Coke

Solution :

$$X \sim Binomial(4\,, 0.95) \quad \leftrightarrow \quad , \quad f_X(x) = \binom{4}{x}(0.95)^x(0.05)^{4-x} \quad , x = 0, 1, \ldots, 4$$

a) PMF table

X	$f(x)$
0	$\binom{4}{0}(0.95)^0\,(0.05)^4 = 0.0000062$
1	$\binom{4}{1}(0.95)^1\,(0.05)^3 = 0.00047$

2	$\binom{4}{2}\ (0.95)^2\ (0.05)^2 = 0.01354$
3	$\binom{4}{3}\ (0.95)^3\ (0.05)^1 = 0.17147$
4	$\binom{4}{4}\ (0.95)^4\ (0.05)^0 = 0.81451$

b) $P(X \geq 2) = P(X = 2) + P(X = 3) + P(X = 4)$
$= 0.01354 = 0.17147 + 0.81451 = 0.99952$

Or $P(X \geq 2) = 1 - P(X < 2) = 1 - \sum_{x=0}^{1} f(x)$

c) $E(X) = \sum_{x=0}^{4} x\, f(x)$
$= 0(0.0000062) + 1(0.00047) + 2(0.01354) + 3(0.17147) + 4(0.81451) = 3.8$

Or $E(X) = np = 4(0.95) = 3.8$

d) $Var(X) = np(1-p) = 4(0.95)(0.05) = 0.19$

Q27) A company installs new central heating furnaces and has found that for 15% of all installations a return visit is needed to make some modifications. **Six** installations were made in a particular week. Assume independence of outcomes for these installations.

a. What is the probability that a return visit was needed in all these cases?
b. What is the probability that a return visit was needed in none of these cases?
c. What is the probability that a return visit was needed in more than one of these cases?

Solution :

X:# of installations need to make some modifications from 6 installations.

$X \sim Binomial(6\,, 0.15) \quad \leftrightarrow \quad \boldsymbol{f_X(x) = \binom{6}{x} (0.15)^x\ (0.85)^{6-x}} \quad , \quad \boldsymbol{x = 0, 1, \dots, 6}$

a) $f_X(6) = p(x = 6) = \binom{6}{6}(0.15)^6(0.85)^0 = 1(0.0000114)(1) = 0.0000114$

b) $f_X(0) = p(x = 0) = \binom{6}{0}(0.15)^0\ (0.85)^6 = 1(1)\ (0.3771) = 0.3771$

c) $P(x > 1) = \sum_{x=2}^{6} \binom{6}{x}(0.15)^x(0.85)^{6-x}$

Or $P(x > 1) = 1 - P(x \leq 1) = 1 - [p(x = 0) + p(x = 1)]$

$= 1 - \left[0.3771 + \binom{6}{1}(0.15)(0.85)^5\right] = 1 - (0.3771 + 0.3993) = 0.2236$

Q28) A fair die is rolled 4 times. Find

a. The probability of obtaining exactly one 6.
b. The probability of obtaining no 6.
c. The probability of obtaining at least one 6.

Solution :

X: The number of times a 6 appears from rolling the dice 4 times.

Each roll is independent and the chance of getting a 6 is 1/6 (success) .

Probability of not getting 6 is $\mathbf{1-\frac{1}{6}=\frac{5}{6}}$ (failure).

$$X\sim Binomial(4\,,1/6) \quad \leftrightarrow \quad \boldsymbol{f_X(x)=\binom{4}{x}\left(\frac{1}{6}\right)^{x}\left(\frac{5}{6}\right)^{4-x}} \quad , \quad \boldsymbol{x=0,1,\dots,4}$$

a) $P(x=1)=\binom{4}{1}\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^{3}=0.386$

b) $P(X=0)=\binom{4}{0}\left(\frac{1}{6}\right)^{0}\left(\frac{5}{6}\right)^{4}=0.482$

c) $P(X\geq 1)=1-\mathrm{P(X=0)}=0.518$

Or $P(X\geq 1)=\sum_{X=1}^{4}\binom{4}{x}\left(\frac{1}{6}\right)^{x}\left(\frac{5}{6}\right)^{4-x}=0.518$

Geometric distribution

P(H) = P(T) = 1/2

Q6) Three people toss a fair coin and the odd man pays for coffee. If the coins all turn up the same, they are tossed again. Find the probability that fewer than 4 tosses are needed. (The odd man is the one with a different outcome from the rest.)

Solution :

Let us consider that a toss the coin 3 times is a trial . Trials are independent.

X: the number of trials needed to get a different outcome from the rest (one tail or head) .

$$Geometric\ dist:\ f_X(x)=P(1-P)^{x-1} \quad ; \quad x=1,2,\dots.$$

To find P :{H,T}*{H,T}*{H,T} = {HHH,HHT,HTH,HTT,THH,THT,TTH,TTT}

P=P(coin of 3 results is different) = P{HHT,HTH,HTT,THH,THT,TTH}

$P=\frac{6}{8}=\frac{3}{4}$

Therefor $P(X<4)=\sum_{x=1}^{3}\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^{x-1}=\frac{63}{64}=0.984375\ \cong 0.9844$

Q7) According to a study published by a group of University of Massachusetts sociologists, **about two thirds** of the 20 million persons in this country who take Valium are women. Assuming this figure to be a valid estimate, find the probability that on a given day **the fifth prescription** written by a doctor for Valium is

a. The first prescribing Valium for a woman.
b. The third prescribing Valium for a woman.

Solution :

a) X: # of prescription of Valium until the **first** woman take Valium .
(Trials are independent)

P=P (women take valium) = 2/3

$$Geometric\ dist:\ f_X(x) = \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^{x-1} \quad ; x = 1,2, ...$$

$$f_X(5) = \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^{5-1} = \frac{2}{243} \quad ; x = 1,2, ...$$

b) Y: # of prescriptions of valium until the **third** woman take Valium.

$$Negative\ Binomial\ dist:\ f_Y(y) = \binom{y-1}{3-1}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^{y-3} \quad ; y = 3,4,5, ...$$

$$f_Y(5) = \binom{5-1}{3-1}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^{5-3} = \binom{4}{2}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^2 = \frac{16}{81}$$

Q8) The probability that a student passes the written test for a private pilot's license is 0.7. Find the probability that the student will pass the test

a. On the third try.

b. Before the fourth try.

Solution :

X: # of try written test until get first passes of the test. (Trials are independent)
P: P(passes written test) = 0.7

$$\text{Geometric distribution: } f_X(x) = (0.7)(0.3)^{x-1} \quad , x = 1,2, ..$$

(a) $P(X = 3) = f_X(3) = (0.7)(0.3)^2 = 0.0630$

(b) $P(X < 4) = P(X \le 3) = \sum_{X=1}^{3} f_X(x) = \sum_{x=1}^{3}(0.7)(0.3)^{x-1} = 0.9730$

Q19) Suppose X has a geometric distribution with p=0.8. Compute the probability of the following events.

a) $X > 3$
b) $4 \le x \le 7$
c) $3 < x \le 5$ or $7 \le x \le 10$

Solution :

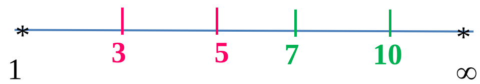
$$f_X(x) = pq^{(x-1)} \quad , \qquad x = 1,2,\dots \quad ; p = 0.8 \ ; q = 0.2$$

a) $P(X > 3) = 1 - P(X \le 3) = 1 - \sum_{X=1}^{3}(0.8)(0.2)^{x-1} \ = 0.008$

b) $P(4 \le X \le 7) = \sum_{X=4}^{7}(0.8)(0.2)^{X-1} = 0.007987200$

c) $P(3 \le X \le 5) \ or \ P(7 \le X \le 10) = P(3 < X \le 5) + P(7 \le X \le 10) - P(\phi) = 0.007987200$

$(3 \le X \le 5) \cap (7 \le X \le 10) = \phi$



Q20) If the probability is 0.75 that an application for a driver's license will pass the road test on any given try, what is the probability that an application will finally pass the test on the fourth try

Solution :

X: the number of tries to pass the test (get the driver's license) ; $P = 0.75 \ \ q = 0.25$

Geometric distribution: $f_X(x) = P(X = x) = pq^{x-1} \quad , \ x = 1,2,\dots$

$$F_X(4) = p(x = 4) = (0.75)(0.25)^3 = 0.0117$$

Q21) Suppose that 30% of the applicants for a certain industrial job have advanced training in computer programming. Applicants are interviewed sequentially and are selected at random from the pool. Find the probability that the first applicant having advanced in programming is found on the fifth interview.

Solution :

X: the number of applicants interviewed to obtain the first applicant with advanced training in programming. $P = 0.3 \ \ q = 0.7$

Geometric distribution where $f_X(x) = P(X = x) = pq^{x-1} \quad , \ x = 1,2,\dots$

$f_X(5) = P(X = 5) = (0.3)(0.7)^4 = 0.07203$

Negative Binomial distribution

Q23) If the probability is 0.40 that a child exposed to a certain contagious disease will catch it, what is the probability that the tenth child exposed to the disease will be the third to catch it.

Solution :

Negative Binomial distribution $f_X(x) = \binom{x-1}{r-1} p^r q^{x-r} \quad , x = r, r+1, r+2, \dots$

$$p = 0.4 \ \ , q = 0.6 \ \ , r = 3$$

X:# of children that they exposed to the disease until the disease catch the third child.

$$P(X = 10) = f_X(10) = \binom{9}{2} (0.4)^3 (0.6)^7 = 0.0645$$

Q24) In an assembly process, the finished items are inspected by a vision sensor, the image data is processed , and a determination is made by computer as to whether or not a unit is satisfactory. If it is assumed that 2% of the units will be rejected, then what is the probability that the thirtieth unit observed will be second rejected unit?

Solution :

Negative Binomial distribution $f_X(x) = \binom{x-1}{r-1} p^r q^{x-r} \quad , x = r, r+1, r+2, \ldots$

$$p = 0.02 \quad , q = 0.98 \quad , r = 2$$

$f_X(30) = P(X = 30) = \binom{29}{1}(0.02)^2(0.98)^{28} = 0.0066$

Poisson distribution

Q12) On average, a certain intersection results in 3 traffic accidents per month.

1. For any given month at this intersection. What is the probability that:

a. Exactly 5 accidents will occur?
b. Less than 3 accidents will occur?
c. At least 2 accidents will occur?

Solution :

$f(x) = \frac{e^{-3}\ 3^x}{x!} \quad ; x = 0,1,2, \ldots$

$\lambda = 3$traffic accidents / month. X: # of traffic accidents in a month

a) Using the poison distribution with x=5 and $\lambda = 3$, we find from table A.2 that $P(X = 5) = \frac{e^{-3}\ 3^5}{5!} = 0.1008$

b) $\text{P}(X < 3) = \text{P}(X \leq 2) = \sum_{x=0}^{2} \frac{e^{-3}\ 3^x}{x!} = 0.4232$

c) $\text{P}(X \geq 2) = 1 - \text{P}(X < 2) = 1 - \sum_{x=0}^{1} \frac{e^{-3}\ 3^x}{x!} = 0.8009$

2. For any given year at this intersection. What is the probability that:

a. Exactly 5 accidents will occur?
b. Less than 3 accidents will occur?
c. At least 2 accidents will occur?

Solution :

Y: # of traffic accidents in a year .

$1\ month \rightarrow \lambda = 3$

$1\ year = 12 month \rightarrow \lambda^* = 3(12) = 36\ \ traffic\ accidents\ /year$

$f(y) = \frac{e^{-36}\ 36^y}{y!} \quad ; x = 0,1,2, \ldots$

a) $P(Y = 5) = f_Y(5) = 1.16877 \times 10^{-10}$

b) $\text{P}(Y < 3) = \text{P}(Y \leq 2) = \sum_{y=0}^{2} \frac{e^{-36}\ 36^y}{y!} = 1.589 \times 10^{-13}$

c) $\text{P}(Y \geq 2) = 1 - \text{P}(Y < 2) \approx 1$

Q13) A secretary makes 2 errors per page, on average. What is the probability that on the next page he or she will make

a. 4 or more errors?
b. No errors?

Solution :

$$f(x) = \frac{e^{-2}\ 2^x}{x!}\ \ ;x = 0,1,2,\ldots$$

$\lambda = 2$ error / page. X: # of error in a page

a) $P(X \geq 4) = 1 - P(X \leq 3) = 1 - \sum_{x=0}^{3} \frac{e^{-2}\ 2^x}{x!} = 0.1429$

b) $P(X = 0) = \frac{e^{-2}\ 2^0}{0!} = 0.1353$

Q14) A certain area of the eastern United States is, on average, hit by 6 hurricanes a year. Find the probability that for a given year that area will be hit by

a. Fewer than 4 hurricanes.
b. Anywhere from 6 to 8 hurricanes.
c. Find the probability that for a given **3 months** that area will be hit by fewer than 4 hurricanes.

Solution :

$$f(x) = \frac{e^{-6}\ 6^x}{x!}\ \ ;x = 0,1,2,\ldots$$

$\lambda = 6$ hurricanes\year . X: # of hit by hurricanes in a year

a) $P(X < 4) = P(X \leq 3) = \sum_{x=0}^{3} \frac{e^{-6}\ 6^x}{x!} = 0.1512$

b) $P(6 \leq X \leq 8) = \sum_{x=6}^{8} \frac{e^{-6}\ 6^x}{x!} = 0.4015$

c) $1\ year = 12 month\ \rightarrow \lambda = 6$

$3\ month\ \rightarrow \lambda^* = \frac{6\times 3}{12} = 1.5$ hit by hurricanes\3 months.

Y: # of hit by hurricanes in 3 months

$f(y) = \frac{e^{-1.5}\ 1.5^y}{y!}\ \ ;x = 0,1,2,\ldots$

$P(Y < 4) = P(Y \leq 3) = \sum_{y=0}^{3} \frac{e^{-1.5}\ 1.5^y}{y!}$

Q29) In a study of a drug -induced anaphylaxis among patients taking rocuronium bromide as part of their anesthesia, Laake and Rottingen found that the occurrence of anaphylaxis followed a Poisson model with =12 incidents per year in Norway .Find

1. The probability that in the next year, among patients receiving rocuronium, exactly three will experience anaphylaxis?
2. The probability that less than two patients receiving rocuronium, in the next year will experience anaphylaxis?
3. The probability that more than two patients receiving rocuronium, in the next two years will experience anaphylaxis?
4. The expected value of patients receiving rocuronium, in the next 6 months who will experience anaphylaxis.
5. The variance of patients receiving rocuronium, in the next year who will experience anaphylaxis.

6. The standard deviation of patients receiving rocuronium, in the next year who will experience anaphylaxis.

Solution :

Poisson distribution

X:# of patients that have anaphylaxis from medication per year .

$\boldsymbol{\lambda = 12/year}$ [average of patients that have anaphylaxis from medication per year]

$$f_X(x) = \frac{e^{-\lambda}\,\lambda^x}{x!} \quad ; \quad x = 0,1,2,\ldots$$

1) $P(X=3) = fx(x) = \frac{e^{-12}\,(12)^3}{3!} = 0.001769533$

2) $P(X<2) = P(X \leq 1) = \sum_{x=0}^{1} \frac{e^{-12}\,(12)^x}{x!} = 0.0000799$

   Or $P(X \leq 1) = P(X=0) + P(X-1) = e^{-12}\left(\frac{12^0}{0!} + \frac{12^1}{1!}\right) = 0.0000799$

3) $P(X>2) = 1 - P(X \leq 2) = 1 - \sum_{x=0}^{2} \frac{e^{-24}\,(24)^x}{x!} = 0.9999$

4) $\mu = E(X) = \lambda = \frac{12}{2} = 6$

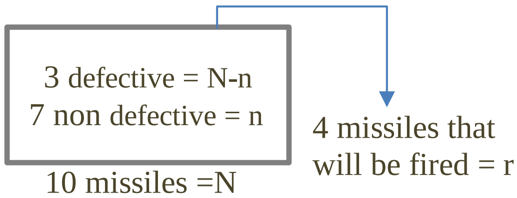
5) $\sigma^2 = var(X) = \lambda = 12$

6) $\sigma = \sqrt{Var(x)} = \sqrt{12}$

Hypergeometric distribution

Q9) From a lot of 10 missiles, 4 are selected at random and fired. If the lot contains 3 defective missiles that will not fire, what is the probability that

a. All 4 will fire?
b. At most 2 will not fire?



Solution :

$$f(k) = P(X = k) = \frac{\binom{n}{k}\binom{N-n}{r-k}}{\binom{N}{r}} \quad ; k = 0,1, \dots \min(r, n) \quad ; \begin{cases} \text{N : population size.} \\ \text{n : no. of success in population.} \\ \text{r : sample size.} \\ k: \textit{no. of success in sample.} \end{cases}$$

a) Probability that all 4 will fire

$$P(X = 4) = \frac{\binom{7}{4}\binom{3}{0}}{\binom{10}{4}} = \frac{1}{6} = 0.1667$$

b) Probability that at most 2 will not fire [2 or less]

$$P(X \leq 2) = \frac{\binom{3}{2}\binom{7}{2}}{\binom{10}{4}} + \frac{\binom{3}{1}\binom{7}{3}}{\binom{10}{4}} + \frac{\binom{3}{0}\binom{7}{4}}{\binom{10}{4}} = \frac{29}{30}$$

$$or\ P(X \leq 2) = 1 - P(\text{3 missles will not fire})$$

$$= 1 - P(X = 3) = 1 - \frac{\binom{3}{3}\binom{7}{1}}{\binom{10}{4}} = \frac{29}{30}$$

Q10) A random committee of size 3 is selected from 4 doctors and 2 nurses. Write a formula for the probability distribution of the random variable X representing the number of doctors on the committee. Find $P(2 \leq X \leq 3)$

Solution :

$$P(2 \leq X \leq 3) = \frac{\binom{4}{2}\binom{2}{1}}{\binom{6}{3}} + \frac{\binom{4}{3}\binom{2}{0}}{\binom{6}{3}} = \frac{4}{5}$$

Q11) A manufacturing company uses an acceptance scheme on production items before they are shipped. The plan is a two-stage one. Boxes of 25 items are readied for shipment, and a sample of 3 items is tested for defectives. If any defectives are found, the entire box is sent back for 100% screening. If no defectives are found, the box is shipped.

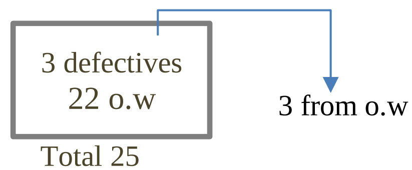
a. What is the probability that a box containing 3 defectives will be shipped?
b. What is the probability that a box containing only 1 defective will be sent back for screening?

Solution :

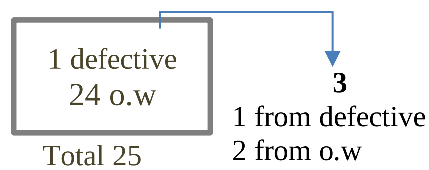
Hypergeometric distribution: $P(X=k)=\frac{\binom{n}{k}\binom{N-n}{r-k}}{\binom{N}{r}}$ $\quad ; k=0,1,\ldots\min(r,n)$

Let x: the number of defective items among the 3 items.

a) $P(X=0)=\frac{\binom{3}{0}\binom{22}{3}}{\binom{25}{3}}=\frac{77}{115}=0.6696$



b) $P(X=1)=\frac{\binom{1}{1}\binom{24}{2}}{\binom{25}{3}}=\frac{3}{25}=0.12$



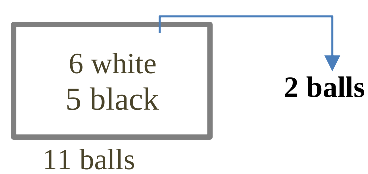
Q25) If **2** balls are randomly drawn from a bowl containing **6** white and **5** black balls, what is the probability that **one** of the drawn balls is white and the other black?

Solution :

Hypergeometric distribution: $P(X=k)=\frac{\binom{n}{k}\binom{N-n}{r-k}}{\binom{N}{r}}$ $\quad ; k=0,1,\ldots\min(r,n)$

X:the number of white balls.

$f(1)=\frac{\binom{6}{1}\binom{5}{1}}{\binom{11}{2}}=0.5455$

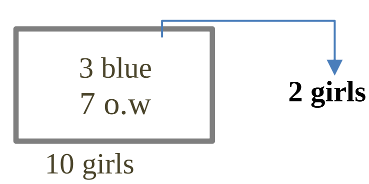


Q26) Of 10 girls in a class, 3 have blue eyes. If **two** of the girls are chosen at random, what is the probability that

a. Both have blue eyes.
b. Neither have blue eyes.
c. At least one has blue eyes.

Solution :

X:the number of girls with blue eyes.



a) $f(2)=\frac{\binom{3}{2}\binom{7}{0}}{\binom{10}{2}}=0.06$

b) $f(0)=\frac{\binom{3}{0}\binom{7}{2}}{\binom{10}{2}}=0.4667$

c) $P(X\geq 1)=\frac{\binom{3}{1}\binom{7}{1}+\binom{3}{2}\binom{7}{0}}{\binom{10}{2}}=0.533$

NOTE : Deleted Questions are 15 , 16 , 22, 30 and 31 .

Summary of Discrete Distributions

random variable	description	density	mean	variance
Geometric	X is the number of independent trials needed to obtain the first success.	$f(x) = p\ q^{x-1} \qquad x = 1,2,\ldots$	$\frac{1}{p}$	$\frac{q}{p^2}$
Binomial	X is the number of successes in *n* independent trial (with replacement).	$f(x) = \binom{n}{x} p^x q^{(n-x)}\ , \quad x = 0,1,2,\ldots$	np	npq
Negative binomial	X is the number of trials needed to obtain r successes.	$f_X(x) = \binom{x-1}{r-1} p^r q^{x-r} \quad , x = r, r+1\ , r+2, \ldots$	$\frac{r}{p}$	$\frac{rq}{p^2}$
Hypergeometric	X is the number of successes in *r* dependent trial (without replacement)	$f(k) = P(X = k) = \frac{\binom{n}{k}\binom{N-n}{r-k}}{\binom{N}{r}}$ $k = 0,1,\ldots \min(r,n)$ N : population size . n : no. of success in population. r : sample size. k : *no. of success in sample.*	$\frac{rn}{N}$	$\left(\frac{rn}{N}\right)\left(\frac{N-n}{N}\right)\left(\frac{N-r}{N-1}\right)$
Poisson	X is the number of random occurrences of some phenomenon in a specified unit of space or time.	$f(x) = \frac{e^{-\lambda}\ \lambda^x}{x!}\ ; x = 0,1,2,\ldots$	λ	λ

P the (constant) probability of success $(0 < p < 1)$

q: the (constant) probability of failure $(q = 1 - p)$

λ: the average number of occurrences of the discrete event in the continuous interval.

The Geometric distribution is a special case of the Negative Binomial distribution such that r = 1.

The Bernoulli distribution is a special case of the Binomial distribution such that n= 1.

*ملاحظة: الرجاء التأكد من عدم وجود أخطاء مطبعية .