Chapter 7

Ultimate Bearing Capacity of Shallow Foundations: Special Cases

Omitted Sections
• Sec 7.6
• Sec 7.10
• Sec 7.12
The ultimate bearing capacity problems described in Chapter 6 assume that:

- The soil supporting the foundation is homogeneous and extends to a great depth below the bottom of the foundation.
- The ground surface is horizontal.

However, that is not true in all cases:

- It is possible to encounter a rigid layer at a shallow depth.
- The soil may be layered and have different shear strength parameters.
- It may be necessary to construct foundations on or near a slope.
- It may be required to design a foundation subjected to uplifting load.

This chapter discusses bearing capacity problems related to these special cases.
Part I

FOUNDATION SUPPORTED BY a SOIL WITH a RIGID BASE AT SHALLOW DEPTH
If a rigid, rough base is located at a depth of $H < D$ below the bottom of the foundation, full development of the failure surface in soil will be restricted. In such a case, the soil failure zone and the development of slip lines at ultimate load will be as shown in the Figure below.

![Figure 7.2](image1.png)

**Figure 7.2**

Failure surface under a rough, continuous foundation with a rigid, rough base located at a shallow depth

![Figure 7.1](image2.png)

**Figure 7.1**

(a) Failure surface under a rough continuous foundation; (b) variation of $D/B$ with soil friction angle $\phi'$
Foundation Supported by a Soil with a Rigid Base at Shallow Depth

\[ q_u = c'N_c^x + qN_q^x + \frac{1}{2}\gamma BN_y^x \]

where

- \( N_c^x, N_q^x, N_y^x \) = modified bearing capacity factors
- \( B \) = width of foundation
- \( \gamma \) = unit weight of soil

For \( H \gg D, N_c^x = N_c, N_q^x = N_q, \) and \( N_y^x = N_y \)

Figure 7.2: Failure surface under a rough, continuous foundation with a rigid, rough base located at a shallow depth
Foundation Supported by a Soil with a Rigid Base at Shallow Depth
Rectangular Foundation on Granular Soil

\[ q_u = q N_q^x F_{q_1}^x + \frac{1}{2} \gamma B N_\gamma^x F_{\gamma_1}^x \]

where \( F_{q_1}^x, F_{\gamma_1}^x \) = modified shape factors.

\[ F_{q_1}^x = 1 - m_1 \left( \frac{B}{L} \right) \]

\[ F_{\gamma_1}^x = 1 - m_2 \left( \frac{B}{L} \right) \]
Square and Circular Foundations on Granular Soil

\[ q_u = qN_q^x + 0.4\gamma BN_{\gamma}^x \quad \text{(square foundation)} \]

\[ q_u = qN_q^x + 0.3\gamma BN_{\gamma}^x \quad \text{(circular foundation)} \]
Foundations on Saturated Clay

\[ q_u = c_u N_e^* + q \]

Table 7.1 Values of \( N_e^* \) for Continuous and Square Foundations (\( \phi = 0 \))

<table>
<thead>
<tr>
<th>( \frac{B}{H} )</th>
<th>( N_e^* ) Square(^a)</th>
<th>( N_e^* ) Continuous(^b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5.43</td>
<td>5.24</td>
</tr>
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<td>3</td>
<td>5.93</td>
<td>5.71</td>
</tr>
<tr>
<td>4</td>
<td>6.44</td>
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<tr>
<td>5</td>
<td>6.94</td>
<td>6.68</td>
</tr>
<tr>
<td>6</td>
<td>7.43</td>
<td>7.20</td>
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<tr>
<td>8</td>
<td>8.43</td>
<td>8.17</td>
</tr>
<tr>
<td>10</td>
<td>9.43</td>
<td>9.05</td>
</tr>
</tbody>
</table>

\(^a\)Buisman’s analysis (1940)
\(^b\)Mandel and Salencon’s analysis (1972)
Example 7.1

A square foundation measuring 1.2 m × 1.2 m is constructed on a layer of sand. We are given that \( D_s = 1 \) m, \( \gamma = 15.5 \) kN/m\(^3\), \( \phi' = 35^\circ \), and \( c' = 0 \). A rock layer is located at a depth of 0.48 m below the bottom of the foundation. Using a factor of safety of 4, determine the gross allowable load the foundation can carry.

**Solution**

From Eq. (5.3),

\[
q_a = qN_q^*q_{st}^* + \frac{1}{2}\gamma BN_r^*r_{st}^*
\]

and we also have

\[
q = 15.5 \times 1 = 15.5 \text{ kN/m}^3
\]

For \( \phi' = 35^\circ \), \( H/B = 0.48/1.2 = 0.4 \), \( N_q^* = 336 \) (Figure 5.4), and \( N_r^* = 138 \) (Figure 5.5), and we have

\[
F_{qs}^* = 1 - m_1 \left( \frac{B}{L} \right)
\]

From Figure 5.6a for \( \phi' = 35^\circ \), \( H/B = 0.4 \). The value of \( m_1 = 0.58 \), so

\[
F_{qs}^* = 1 - (0.58)(1.2/1.2) = 0.42
\]

Similarly,

\[
F_{rs}^* = 1 - m_2 (B/L)
\]

From Figure 5.6b, \( m_2 = 0.6 \), so

\[
F_{rs}^* = 1 - (0.6)(1.2/1.2) = 0.4
\]

Hence,

\[
q_a = (15.5)(336)(0.42) + (1/2)(15.5)(1.2)(138)(0.4) = 2700.72 \text{ kN/m}^2
\]

and

\[
Q_{at} = \frac{q_a B^2}{FS} = \frac{(2700.72)(1.2 \times 1.2)}{4} = 972.3 \text{ kN}
\]
EXAMPLE 7.1
EXAMPLE 7.1

For $H/B = 0.1$, the values of $m_1$ and $m_2$ are 0.58 and 0.4, respectively.
Example 7.2

Consider a square foundation 1 m × 1 m in plan located on a saturated clay layer underlain by a layer of rock. Given:

Clay: $c_u = 72$ kN/m²
Unit weight: $\gamma = 18$ kN/m³
Distance between the bottom of foundation and the rock layer $= 0.25$ m
$D_f = 1$ m

Estimate the gross allowable bearing capacity of the foundation. Use FS = 3.

Solution

From Eq. (5.10),

$$q_u = 5.14 \left( 1 + \frac{0.5 \frac{B}{H} - 0.707}{5.14} \right) c_u + q$$

For $B/H = 1/0.25 = 4$; $c_u = 72$ kN/m²; and $q = \gamma D_f = (18)(1) = 18$ kN/m³.

$$q_u = 5.14 \left[ 1 + \frac{(0.5)(4) - 0.707}{5.14} \right] 72 + 18 = 481.2 \text{ kN/m}^2$$

$$q_{all} = \frac{q_u}{FS} = \frac{481.2}{3} = 160.4 \text{ kN/m}^2$$
Part II
FOUNDATIONS ON LAYERED MEDIUM
In nature, soil is generally non-homogeneous with mixtures of sand, silt and clay in different proportions.

In the analysis, an average profile of such soils is normally considered.

However, if soils are found in distinct layers of different compositions and strength characteristics, the assumption of homogeneity to such soils is not strictly valid if the failure surface cuts across boundaries of such layers.

Further, we may come across the upper layer strong and the lower layer weak or vice versa.

The bearing capacity equations presented in Chapter 4 involve cases in which the soil supporting the foundation is homogeneous and extends to a considerable depth.
The cohesion, angle of friction, and unit weight of soil were assumed to remain constant for the bearing capacity analysis. However, in practice, layered soil profiles are often encountered.

In such instances, the failure surface at ultimate load may extend through two or more soil layers, and a determination of the ultimate bearing capacity in layered soils can be made in only a limited number of cases.

No simple, satisfactory, analytical method is currently available to determine the bearing capacity of layered soils.

The present analysis is limited to a system of two distinct soil layers. For a footing located in the upper layer at a depth D, below the ground level, the failure surfaces at ultimate load may either lie completely in the upper layer or may cross the boundary of the two layers.
Foundations on Layered Medium

Cases

Case 1. Footing on layered clays (all $\phi = 0$) as in Fig. 4-5a.
   a. Top layer weaker than lower layer ($c_1 < c_2$)
   b. Top layer stronger than lower layer ($c_1 > c_2$)

Case 2. Footing on layered $\phi$–$c$ soils with $a, b$ same as case 1.

Case 3. Footing on layered sand and clay soils as in Fig. 4-5b.
   a. Sand overlying clay
   b. Clay overlying sand
Foundations on Layered Clay \((\phi = 0)\)

For undrained loading \((\phi = 0\) condition) :

let \(c_u(1)\) = shear strength of the upper clay layer

\(c_u(2)\) = shear strength of the lower clay layer

\[
q_u = c_u(1)N_c F_{cs} F_{cd} + q
\]

The relationships for \(F_{cs}\) and \(F_{cd}\) given in Table 6.3

The variation of \(N_c\) is given in the Figure

If the lower layer of clay is softer than the top one \((c_u(2)/c_u(1) < 1)\), the value of \((N_c)\) is lower than when the soil is not layered \((c_u(2)/c_u(1) = 1)\).

This means that the ultimate bearing capacity is reduced by the presence of a softer clay layer below the top layer.
Weaker Layer underlain by Stronger Layer ($\phi = 0$)

Ultimate bearing capacity of a foundation supported by a weaker clay layer $[c_{u(1)}]$ underlain by a stronger clay layer $[c_{u(2)}]$ i.e $(c_{u(1)}/c_{u(2)} < 1)$:

$$q_u = c_{u(1)}mN_cF_{cs}F_{cd} + q$$

where

- $N_c = \begin{cases} 5.14 \text{ for continuous foundation} \\ 6.17 \text{ for square or circular foundation} \end{cases}$
- $F_{cs} = \text{shape factor}$
- $F_{cd} = \text{depth factor}$
- $m = f\left[\frac{c_{u(1)}}{c_{u(2)}} \cdot \frac{H}{B}, \frac{B}{L}\right]$  

### Table 7.3

Variation of $m$ [Equation (5.12)] for Square Foundation ($B/L = 1$)

<table>
<thead>
<tr>
<th>$c_{u(1)}/c_{u(2)}$</th>
<th>$H/B$</th>
<th>$\geq 0.25$</th>
<th>$0.125$</th>
<th>$0.083$</th>
<th>$0.063$</th>
<th>$0.05$</th>
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<tr>
<td>1</td>
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<td>1</td>
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<td>1</td>
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<td>1</td>
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<tr>
<td>0.667</td>
<td>1</td>
<td>1.028</td>
<td>1.052</td>
<td>1.075</td>
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<td>1</td>
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<tr>
<td>0.5</td>
<td>1</td>
<td>1.047</td>
<td>1.091</td>
<td>1.131</td>
<td>1.167</td>
<td>1.177</td>
</tr>
<tr>
<td>0.333</td>
<td>1</td>
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<td>1.143</td>
<td>1.207</td>
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<td>1.177</td>
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<td>1.254</td>
<td>1.376</td>
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</table>

Based on Vesic (1975)

### Table 7.2

Variation of $m$ [Equation (5.12)] for Continuous Foundation ($B/L \leq 0.2$)

<table>
<thead>
<tr>
<th>$c_{u(1)}/c_{u(2)}$</th>
<th>$H/B$</th>
<th>$\geq 0.5$</th>
<th>$0.25$</th>
<th>$0.167$</th>
<th>$0.125$</th>
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<td>1</td>
</tr>
<tr>
<td>0.667</td>
<td>1</td>
<td>1.033</td>
<td>1.064</td>
<td>1.088</td>
<td>1.109</td>
<td>1</td>
</tr>
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<td>0.5</td>
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<td>1.107</td>
<td>1.152</td>
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<td>0.333</td>
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<td>1.154</td>
<td>1.302</td>
<td>1.446</td>
<td>1.584</td>
<td>1.584</td>
</tr>
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</table>

Based on Vesic (1975)
Example 7.3

A foundation 1.5 m × 1 m is located at a depth (D_f) of 1 m in a clay. A stouter clay layer is located at a depth (H) of 1 m measured from the bottom of the foundation. Given:

- For top clay layer,
  - Undrained shear strength = 120 kN/m²
  - Unit weight = 16.8 kN/m³
- For bottom clay layer,
  - Undrained shear strength = 48 kN/m²
  - Unit weight = 16.2 kN/m³

Determine the gross allowable load for the foundation with a factor of safety of 4. Use Eq. (5.11).

**Solution**

From Eq. (5.11),

\[ q_a = c_{at(1)}N_cF_{cr}F_{cd} + q \]

\[ c_{at(1)} = 120 \text{ kN/m}^2 \]

\[ q = \gamma D_f = (16.8)(1) = 16.8 \text{ kN/m}^2 \]

\[ \frac{c_{at(2)}}{120} = 0.4; \frac{H}{B} = \frac{1}{1} = 1 \]

From Figure 5.8b, for \( H/B = 1 \) and \( c_{at(2)}/c_{at(1)} = 0.4 \), the value of \( N_c \) is equal to 4.6. From Table 4.3,

\[ F_{cr} = 1 + \left( \frac{B}{L} \right) \left( \frac{N_q}{N_c} \right) = 1 + \left( \frac{1}{1.5} \right) \left( \frac{4.6}{4.6} \right) = 1.145 \]

\[ F_{cd} = 1 + 0.4 \frac{D_f}{B} = 1 + 0.4 \left( \frac{1}{1} \right) = 1.4 \]

Thus,

\[ q_a = (120)(4.6)(1.145)(1.4) + 16.8 = 884.8 + 16.8 = 901.6 \text{ kN/m}^2 \]

So

\[ q_{at} = \frac{q_a}{FS} = \frac{901.6}{4} = 225.4 \text{ kN/m}^2 \]

Total allowable load = \( (q_{at})(B \times L) = (225.4)(1 \times 1.5) = 338.1 \text{ kN} \)
EXAMPLE 7.3

\[ N_c = 4.6 \]

\[ \frac{c_u(2)}{c_u(1)} = \frac{48}{120} = 0.4; \frac{H}{B} = \frac{1}{1} = 1 \]
If the depth $H$ is relatively small compared with the foundation width $B$, a punching shear failure will occur in the top soil layer, followed by a general shear failure in the bottom soil layer (Figure a).

If the depth $H$ is relatively large, then the failure surface will be completely located in the top soil layer, which is the upper limit for the ultimate bearing capacity (Figure b).
The variation of $K_s$ with $q_2/q_1$ and $\phi_1$ is shown in Figure.
The variation of $c_d/c_1$ with $q_2/q_1$ is shown in Figure.

If the height $H$ is relatively large, then the failure surface in soil will be completely located in the stronger upper-soil layer. For this case,

\[ q_u = q_f = c_1'N_{c(1)} + qN_{q(1)} + \frac{1}{2}\gamma_1BN_{\gamma(1)}. \]  

(b)

Combining Eqs. (a) and (b) yields

\[ q_u = q_b + \frac{2c_a'\gamma_1H}{B} \left( 1 + \frac{2D_f}{H} \right) \frac{K_s\tan \phi_1'}{B} - \gamma_1H \leq q_f. \]
Stronger Layer underlain by Weaker Layer ($c' - \phi' \text{ soil}$)

For rectangular foundations

$$q_u = q_b + \left(1 + \frac{B}{L}\right)\left(\frac{2c'_d H}{B}\right) + \gamma_1 H^2 \left(1 + \frac{B}{L}\right) \left(1 + \frac{2D_f}{H}\right) \left(\frac{K_s \tan \phi'_1}{B}\right) - \gamma_1 H \leq q_f$$

where

$$q_b = c'_2 N_{c(2)} F_{cs(2)} + \gamma_1 (D_f + H) N_{q(2)} F_{qs(2)} + \frac{1}{2} \gamma_2 B N_{\gamma(2)} F_{\gamma(2)}$$

and

$$q_f = c'_1 N_{c(1)} F_{cs(1)} + \gamma_1 D_f N_{q(1)} F_{qs(1)} + \frac{1}{2} \gamma_1 B N_{\gamma(1)} F_{\gamma(1)}$$

in which

$F_{cs(1)}, F_{qs(1)}, F_{\gamma(1)} = \text{shape factors with respect to top soil layer (Table 4.3)}$

$F_{cs(2)}, F_{qs(2)}, F_{\gamma(2)} = \text{shape factors with respect to bottom soil layer (Table 4.3)}$
Top layer is strong sand and bottom layer is saturated soft clay $\phi_2 = 0$

$$q_b = \left(1 + 0.2 \frac{B}{L}\right)5.14c_{u(2)} + \gamma_1(D_f + H)$$

and

$$q_t = \gamma_1D_fN_{q(1)}F_{q(1)} + \frac{1}{2}\gamma_1BN_{\gamma(1)}F_{\gamma(1)}$$

Hence,

$$q_u = \left(1 + 0.2 \frac{B}{L}\right)5.14c_{u(2)} + \gamma_1H^2\left(1 + \frac{B}{L}\right)\left(1 + \frac{2D_f}{H}\right)\frac{K_s \tan \phi'}{B}$$

$$+ \gamma_1D_f \leq \gamma_1D_fN_{q(1)}F_{q(1)} + \frac{1}{2}\gamma_1BN_{\gamma(1)}F_{\gamma(1)}$$

where $c_{u(2)}$ = undrained cohesion.

For a determination of $K_s$ from Figure 5.10,

$$\frac{q_2}{q_1} \approx \frac{c_{u(2)}N_{c(2)}}{\frac{1}{2}\gamma_1BN_{\gamma(1)}} \approx \frac{5.14c_{u(2)}}{0.5\gamma_1BN_{\gamma(1)}}$$
Stronger Layer underlain by Weaker Layer ($c' - \phi' \text{ soil}$)

Top layer is stronger sand and bottom layer is weaker sand ($c'_1 = 0$, $c'_2 = 0$).

$$q_u = \left[ \gamma_1(D_f + H)N_{q(2)}F_{qs(2)} + \frac{1}{2} \gamma_2BN_{\gamma(2)}F_{\gamma(2)} \right]$$

$$+ \gamma_1H^2\left( 1 + \frac{B}{L} \right)\left( 1 + \frac{2D_f}{H} \right) \frac{K_s \tan \phi'_1}{B} - \gamma_1H \leq q_t$$

where

$$q_t = \gamma_1D_fN_{q(1)}F_{qs(1)} + \frac{1}{2} \gamma_1BN_{\gamma(1)}F_{\gamma(1)}$$

Then

$$\frac{q_2}{q_1} = \frac{\frac{1}{2} \gamma_2BN_{\gamma(2)}}{\frac{1}{2} \gamma_1BN_{\gamma(1)}} = \frac{\gamma_2N_{\gamma(2)}}{\gamma_1N_{\gamma(1)}}$$
Stronger Layer underlain by Weaker Layer ($c' - \phi' \text{ soil}$)

Top layer is stronger saturated clay and bottom layer is weaker saturated clay ($\phi_1 = \phi_2 = 0$)

\[ q_u = \left(1 + 0.2 \frac{B}{L}\right)5.14c_{u(2)} + \left(1 + \frac{B}{L}\right)\left(\frac{2c_aH}{B}\right) + \gamma_l D_f \leq q_t \]

where

\[ q_t = \left(1 + 0.2 \frac{B}{L}\right)5.14c_{u(1)} + \gamma_l D_f \]

and $c_{u(1)}$ and $c_{u(2)}$ are undrained cohesions. For this case,

\[ \frac{q_2}{q_1} = \frac{5.14c_{u(2)}}{5.14c_{u(1)}} = \frac{c_{u(2)}}{c_{u(1)}} \]
EXAMPLE 7.4

Refer to Figure 7.9 and consider the case of a continuous foundation with $B = 2$ m, $D_f = 1.2$ m, and $H = 1.5$ m. The following are given for the two soil layers:

**Top sand layer:**
- Unit weight $\gamma_1 = 17.5$ kN/m$^3$
- $\phi'_1 = 40^\circ$
- $c'_1 = 0$

**Bottom clay layer:**
- Unit weight $\gamma_2 = 16.5$ kN/m$^3$
- $\phi'_2 = 0$
- $c'_{a(2)} = 30$ kN/m$^2$

Determine the gross ultimate load per unit length of the foundation.

**Solution**

For this case, Eqs. (5.27) and (5.28) apply. For $\phi'_1 = 40^\circ$, from Table 4.2, $N_\gamma = 109.41$ and

$$
q_2 = \frac{c_{a(2)}N_{c(2)}}{0.5\gamma_1BN_{\gamma(1)}} = \frac{(30)(5.14)}{(0.5)(17.5)(2)(109.41)} = 0.081
$$

From Figure 5.10, for $c_{a(2)}N_{c(2)}/0.5\gamma_1BN_{\gamma(1)} = 0.081$ and $\phi'_1 = 40^\circ$, the value of $K_f = 2.5$. Equation (5.27) then gives

$$
q_a = \left[1 + (0.2)\left(\frac{B}{L}\right)\right]5.14c_{a(2)} + \left(1 + B\right)\gamma_1H^2\left(1 + \frac{2D_f}{H}\right)K_f\frac{\tan \phi'_1}{B} + \gamma_1D_f
$$

$$
= [1 + (0.2)(0)][(5.14)(30) + (1 + 0)(17.5)(1.5)^2]
\times \left[1 + \frac{(2)(1.2)}{1.5} \frac{\tan 40}{2.0} + (17.5)(1.2)\right]
$$

$$
= 154.2 + 107.4 + 21 = 282.6 \text{ kN/m}^2
$$

Again, from Eq. (5.26),

$$
q_t = \gamma_tD_fN_{q(1)}F_{q(1)} + \frac{1}{2} \gamma_tBN_{q(1)}F_{q(1)}
$$

From Table 4.2, for $\phi'_1 = 40^\circ$, $N_\gamma = 109.4$ and $N_q = 64.20$.

From Table 4.3,

$$
F_{q(1)} = 1 + \left(\frac{B}{L}\right)\tan \phi'_1 = 1 + (0)\tan 40 = 1
$$

and

$$
F_{q(1)} = 1 - 0.4\frac{B}{L} = 1 - (0.4)(0) = 1
$$

so that

$$
q_t = (17.5)(1.2)(64.20)(1) + \left(\frac{1}{2}\right)(17.5)(2)(109.4)(1) = 3262.7 \text{ kN/m}^2
$$

Hence,

$$
q_a = 282.6 \text{ kN/m}^2
$$

$$
Q_a = (282.6)(B) = (282.6)(2) = 565.2 \text{ kN/m}
$$
EXAMPLE 7.4

Figure 7.10

Meyerhof and Hanna’s punching shear coefficient $K_s$
Example 7.5

A foundation 1.5 m × 1 m is located at a depth, $D_f$, of 1 m in a stronger clay. A softer clay layer is located at a depth, $H$, of 1 m measured from the bottom of the foundation. For the top clay layer,

Undrained shear strength = 120 kN/m²
Unit weight = 16.8 kN/m³

and for the bottom clay layer,

Undrained shear strength = 48 kN/m²
Unit weight = 16.2 kN/m³

Determine the gross allowable load for the foundation with an FS of 4. Use Eqs. (5.32), (5.33), and (5.34).

Solution

For this problem, Eqs. (5.32), (5.33), and (5.34) will apply, or

$$q_u = \left(1 + \frac{0.2B}{L}\right)5.14c_u(t) + \left(1 + \frac{B}{L}\right)\left(\frac{2c_u H}{B}\right) + \gamma_i D_f$$

$$\leq \left(1 + \frac{0.2B}{L}\right)5.14c_u(t) + \gamma_i D_f$$

Given:

$B = 1$ m
$H = 1$ m
$D_f = 1$ m
$L = 1.5$ m
$\gamma_i = 16.8$ kN/m³

From Figure 5.11, $c_u(t_2)/c_u(t_1) = 48/120 = 0.4$, the value of $c_u/c_u(t_1) \approx 0.9$, so

$c_u = (0.9)(120) = 108$ kN/m²

$$q_u = \left[1 + (0.2)\left(\frac{1}{1.5}\right)\right](5.14)(48) + \left(1 + \frac{1.5}{1}\right)\left(\frac{2(108)(1)}{1}\right) + (16.8)(1)$$

$$= 279.6 + 360 + 16.8 = 656.4 \text{ kN/m}^2$$

Check: From Eq. (5.33),

$$q_t = \left[1 + (0.2)\left(\frac{1}{1.5}\right)\right](5.14)(120) + (16.8)(1)$$

$$= 699 + 16.8 = 715.8 \text{ kN/m}^2$$

Thus $q_u = 656.4 \text{ kN/m}^2$ (that is, the smaller of the two values calculated above) and

$$q_{all} = \frac{q_u}{FS} = \frac{656.4}{4} = 164.1 \text{ kN/m}^2$$

The total allowable load is

$$(q_{all})(1 \times 1.5) = 246.15 \text{ kN}$$

Note: This is the same problem as in Example 5.3. The allowable load is about 40% lower than that calculated in Example 5.3. This is due to the failure surface in the soil assumed at the ultimate load.
Variation of $c'_a/c'_1$ with $q_2/q_1$ based on the theory of Meyerhof and Hanna (1978)
When a foundation is supported by a weaker soil layer underlain by a stronger layer, the ratio of $q_2/q_1$ will be greater than one.

If $H/B$ is relatively small, the failure surface in soil at ultimate load will pass through both soil layers.

However, for larger $H/B$ ratios, the failure surface will be fully located in the top, weaker soil layer.
Weaker Layer underlain by Stronger Layer (c’- φ’ soil)

The ultimate bearing capacity:

\[ q_u = q_t + (q_b - q_t) \left( \frac{H}{D} \right)^2 \geq q_t \]  \hspace{1cm} (5.35)

where

\[ D = \text{depth of failure surface beneath the foundation in the thick bed of the upper weaker soil layer} \]
\[ q_t = \text{ultimate bearing capacity in a thick bed of the upper soil layer} \]
\[ q_b = \text{ultimate bearing capacity in a thick bed of the lower soil layer} \]

So

\[ q_t = c'_1 N_{c(1)} F_{cs(1)} + \gamma_1 D_f N_{q(1)} F_{qs(1)} + \frac{1}{2} \gamma_1 B N_{\gamma(1)} F_{\gamma(1)} \]  \hspace{1cm} (5.36)

and

\[ q_b = c'_2 N_{c(2)} F_{cs(2)} + \gamma_2 D_f N_{q(2)} F_{qs(2)} + \frac{1}{2} \gamma_2 B N_{\gamma(2)} F_{\gamma(2)} \]  \hspace{1cm} (5.37)

where

\[ N_{c(1)}, N_{q(1)}, N_{\gamma(1)} = \text{bearing capacity factors corresponding to the soil friction angle } \phi'_1 \]
\[ N_{c(2)}, N_{q(2)}, N_{\gamma(2)} = \text{bearing capacity factors corresponding to the soil friction angle } \phi'_2 \]
\[ F_{cs(1)}, F_{qs(1)}, F_{\gamma(1)} = \text{shape factors corresponding to the soil friction angle } \phi'_1 \]
\[ F_{cs(2)}, F_{qs(2)}, F_{\gamma(2)} = \text{shape factors corresponding to the soil friction angle } \phi'_2 \]

Meyerhof and Hanna (1978) suggested that

- \( D \approx B \) for loose sand and clay
- \( D \approx 2B \) for dense sand
Example 7.6

Refer to Figure 5.12a. For a layered saturated-clay profile, given: $L = 1.83$ m, $B = 1.22$ m, $D_y = 0.91$ m, $H = 0.61$ m, $\gamma_1 = 17.29$ kN/m$^2$, $\phi_1 = 0$, $c_{u(1)} = 57.5$ kN/m$^2$, $\gamma_2 = 19.65$ kN/m$^2$, $\phi_2 = 0$, and $c_{u(2)} = 119.79$ kN/m$^2$. Determine the ultimate bearing capacity of the foundation.

**Solution**

From Eqs. (5.18) and (5.19),

$$q_2 = \frac{c_{u(2)}N_c}{c_{u(1)}N_c} = \frac{c_{u(2)}}{c_{u(1)}} = \frac{119.79}{57.5} = 2.08 > 1$$

So, Eq. (5.35) will apply.

From Eqs. (5.36) and (5.37) with $\phi_1 = \phi_2 = 0$,

$$q_i = \left(1 + \frac{0.2B}{L}\right)N_c c_{u(1)} + \gamma_f D_y$$

$$= \left[1 + (0.2)\left(\frac{1.22}{1.83}\right)\right](5.14)(57.5) + (0.91)(17.29) = 334.96 + 15.73 = 350.69 \text{ kN/m}^2$$

and

$$q_h = \left(1 + \frac{0.2B}{L}\right)N_c c_{u(2)} + \gamma_f D_y$$

$$= \left[1 + (0.2)\left(\frac{1.22}{1.83}\right)\right](5.14)(119.79) + (0.91)(19.65)$$

$$= 697.82 + 17.88 = 715.7 \text{ kN/m}^2$$

From Eq. (5.35),

$$q_u = q_i + (q_h - q_i)\left(\frac{H}{D}\right)^2$$

$D \approx B$

$$q_u = 350.69 + (715.7 - 350.69)\left(\frac{0.61}{1.22}\right)^2 \approx 442 \text{ kN/m}^2 > q_i$$

Hence,

$$q_u = 442 \text{ kN/m}^2$$
Part III
Closely Spaced Foundations
Effect on Ultimate Bearing Capacity
The preceding analysis were concerned with the bearing capacity of a single footing.

There are, however, conditions in engineering practice where footings are placed so close to each other that their zones of action overlap.

If foundations are placed close to each other with similar soil conditions, the ultimate bearing capacity of each foundation may change due to the interference effect of the failure surface in the soil.
Closely Spaced Foundations—Effect on Ultimate Bearing Capacity

Stuart (1962)

Assumptions for the failure surface in granular soil under two closely spaced rough continuous foundations

\[ \alpha_1 = \phi', \alpha_2 = 45 - \phi'/2, \alpha_3 = 180 - 2\phi' \]

Case I

If the center-to-center spacing of the two foundations is \( x \geq x_1 \), the rupture surface in the soil under each foundation will not overlap.

So the ultimate bearing capacity of each continuous foundation can be given by Terzaghi

For \( (c' = 0) \)

\[ q_u = qN_q + \frac{1}{2}\gamma BN_{\gamma} \]

Where \( N_q, N_{\gamma} = \) Terzaghi’s bearing capacity factors (Table 4.1).
**Case II.**
If the center-to-center spacing of the two foundations \((x = x_2 < x_1)\) are such that the Rankine passive zones just overlap, then the magnitude of \(q_u\) will still be given by Eq. of Case I. However, the foundation settlement at ultimate load will change (compared to the case of an isolated foundation).

\[
q_u = qN_q + \frac{1}{2} \gamma BN_\gamma
\]
**Case III**

This is the case where the center-to-center spacing of the two continuous foundations is $x = x_3 < x_2$. Note that the triangular wedges in the soil under the foundations make angles of $180^\circ - 2\phi'$ at points $d_1$ and $d_2$. The arcs of the logarithmic spirals $d_1 g_1$ and $d_1 e$ are tangent to each other at $d_1$. Similarly, the arcs of the logarithmic spirals $d_2 g_2$ and $d_2 e$ are tangent to each other at $d_2$. For this case, the ultimate bearing capacity of each foundation can be given as

$$q_u = q N_q \zeta_q + \frac{1}{2} \gamma B N_\gamma \zeta_\gamma$$

where $\zeta_q$, $\zeta_\gamma = \text{efficiency ratios}$
Closely Spaced Foundations—Effect on Ultimate Bearing Capacity

\[ q_u = q N_q \zeta_q + \frac{1}{2} \gamma B N_\gamma \zeta_\gamma \]

where \( \zeta_q, \zeta_\gamma \) = efficiency ratios
**Closely Spaced Foundations—Effect on Ultimate Bearing Capacity**

**Case IV.**
If the spacing of the foundation is further reduced such that \( x = x_4 < x_3 \), blocking will occur and the pair of foundations will act as a single foundation. The soil between the individual units will form an inverted arch which travels down with the foundation as the load is applied. When the two foundations touch, the zone of arching disappears and the system behaves as a single foundation with a width equal to \( 2B \). The ultimate bearing capacity for this case can be given by Eq. of Case I, with \( B \) being replaced by \( 2B \) in the second term.

\[
q_u = qN_q + \frac{1}{2} \gamma BN_y
\]

The ultimate bearing capacity of two continuous foundations spaced close to each other may increase since the efficiency ratios are greater than one. However, when the closely spaced foundations are subjected to a similar load per unit area, the settlement \( S_e \) will be larger when compared to that for an isolated foundation.
Closely Spaced Foundations—Effect on Ultimate Bearing Capacity

Case I

\[ x \geq x_1, \]

Case II

\[ (x = x_2 < x_1) \]

Case III

\[ x = x_3 < x_2 \]

Case IV

\[ x = x_4 < x_3, \]

Terzaghi

(Note: \( \alpha_1 = \phi', \alpha_2 = 45 - \phi'/2, \alpha_3 = 180 - 2\phi' \))
PART IV
FOOTING ON OR ADJACENT TO A SLOPE
TWO CASES

Foundations on Top of a Slope

Foundations on a Slope

CASE I

CASE II
The ultimate bearing capacity for *continuous foundations*:

\[ q_u = c'N_{cq} + \frac{1}{2} \gamma B N_{\gamma q} \]

For purely granular soil, \( c' = 0 \), thus,

\[ q_u = \frac{1}{2} \gamma B N_{\gamma q} \]

Again, for purely cohesive soil, \( \phi = 0 \) (the undrained condition); hence,

\[ q_u = c_u N_{cq} \]

where \( c_u = \) undrained cohesion.

![Figure 7.19](image1)

Shallow foundation on top of a slope

![Figure 7.20](image2)

Meyerhof’s bearing capacity factor \( N_{\gamma q} \) for granular soil \( (c' = 0) \)
Weaker Layer underlain by Stronger Layer ($c'-\phi'$ soil)

The following points need to be kept in mind in determining $N_{cq}$:

1. The term

   \[ N_t = \frac{\gamma H}{c_u} \]

   is defined as the stability number.

2. If $B<H$, use the curves for $N_s = 0$.

3. If $B\geq H$, use the curves for the calculated stability number $N_s$.
Example 7.8

In a shallow continuous foundation in a clay, the following data are given: \( B = 1.2 \text{ m} \); \( D_f = 1.2 \text{ m} \); \( b = 0.8 \text{ m} \); \( H = 6.2 \text{ m} \); \( \beta = 30^\circ \); unit weight of soil = 17.5 kN/m\(^3\); \( \phi = 0^\circ \); and \( c_u = 50 \text{ kN/m}^2 \). Determine the gross allowable bearing capacity with a factor of safety FS = 4.

Solution
Since \( B < H \), we will assume the stability number \( N_s = 0 \). From Eq. (5.43),

\[
q_u = c_u N_{eq}
\]

We are given that

\[
\frac{D_f}{B} = \frac{1.2}{1.2} = 1
\]

and

\[
\frac{b}{B} = \frac{0.8}{1.2} = 0.67
\]

For \( \beta = 30^\circ \), \( D_f/B = 1 \) and \( b/B = 0.67 \), Figure 5.21 gives \( N_{eq} = 6.3 \). Hence,

\[
q_u = (50)(6.3) = 315 \text{ kN/m}^2
\]

and

\[
q_{ul} = \frac{q_u}{FS} = \frac{315}{4} = 78.8 \text{ kN/m}^2
\]
\[
\beta = 30^\circ
\]

\[
N_{cq} = 6.3
\]

\[
\frac{D_f}{B} = \frac{1.2}{1.2} = 1
\]

\[
N_s = ???
\]

1. The term

\[
N_s = \frac{\gamma H}{c_u}
\]

is defined as the stability number.

2. If \( B < H \), use the curves for \( N_s = 0 \).

3. If \( B \geq H \), use the curves for the calculated stability number \( N_s \).

\[
B = 1.2 \text{ m} \quad \Rightarrow \quad B < H \quad \Rightarrow \quad N_s = 0
\]

\[
H = 6.2 \text{ m}
\]

\[
\frac{b}{B} = \frac{0.8}{1.2} = 0.67
\]
Example 7.9

Figure 7.22 shows a continuous foundation on a slope of a granular soil. Estimate the ultimate bearing capacity.

Foundation on a granular slope

Solution
For granular soil \( c' = 0 \), from Eq. (5.42),

\[
q_u = \frac{1}{2} \gamma B N_{yq}
\]

We are given that \( b/B = 2/1.5 = 1.33 \), \( D_f/B = 1.5/1.5 = 1 \), \( \phi' = 30^\circ \), and \( \beta = 30^\circ \).

From Figure 5.20, \( N_{yq} \approx 41 \). So,

\[
q_u = \frac{1}{2} (15.5)(1.5)(41) = 476.6 \text{ kN/m}^2
\]
We are given that $b/B = 2/1.5 = 1.33$, $D_f/B = 1.5/1.5 = 1$, $\phi' = 30^\circ$, and $\beta = 30^\circ$. 
Bearing Capacity of Foundations on a Slope

\[ q_u = c_u N_{cq} \] (for purely cohesive soil, that is, \( \phi = 0 \))

\[ q_a = \frac{1}{2} \gamma B \sqrt{q_{ps}} \] (for granular soil, that is \( c' = 0 \))

\[ \phi = 0 \]

\[ N_{cqs} \]

\[ c' = 0 \]

\[ N_{\gamma q_{ps}} \]

Figure 7.24 Variation of \( N_{cq} \) with \( \beta \).
(Note: \( N_c = \gamma H/c_a \))
\[ N_{cqs} \]

\[ N_s = \frac{C_u}{\gamma H} \]
Remarks

- The $N_{c qs}$ and $N_{\gamma qs}$ factors decrease with greater inclination of slope.

- For inclination of slopes used in practice ($\beta < 30^\circ$), the decrease in bearing capacity is small in the case of clays but is considerable for sand and gravel slopes.
Part V
Foundations on Rock
Foundations on Rock

- With the exception of a few porous limestone and volcanic rocks and some shales, the strength of bedrock in situ will be greater than the compressive strength of the foundation concrete.

- This statement may not be true if the rock is in a badly fractured, loose state where considerable relative slip between rock fragments can occur.

- Settlement is more often of concern than is the bearing capacity, and most test effort is undertaken to determine the in situ deformation modulus $E$ and Poisson's ratio so that some type of settlement analysis can be made.

- It is common to use building code values for the allowable bearing capacity of rocks.
Foundations on Rock

\[ q_u = c'N_c + qN_q + 0.5\gamma BN_\gamma \]

\[ N_c = 5 \tan^4 \left( 45 + \frac{\phi'}{2} \right) \]
\[ N_q = \tan^6 \left( 45 + \frac{\phi'}{2} \right) \]
\[ N_\gamma = N_q + 1 \]

\[ q_{ac} = 2c'\tan \left( 45 + \frac{\phi'}{2} \right) \]

where

- \( q_{ac} \) = unconfined compression strength of rock
- \( \phi' \) = angle of friction

\[ q_{u(modified)} = q_u(RQD)^2 \]
Example 7.13

Refer to Figure 5.32. A square column foundation is to be constructed over siltstone.

Given:
Foundation: \( B \times B = 2.5 \text{ m} \times 2.5 \text{ m} \)
\( D_f = 2 \text{ m} \)
Soil:
\( \gamma = 17 \text{ kN/m}^3 \)
Siltstone:
\( c' = 32 \text{ MN/m}^2 \)
\( \phi' = 31^\circ \)
\( \gamma = 25 \text{ kN/m}^3 \)
RDQ = 50%

Estimate the allowable load-bearing capacity. Use FS = 4. Also, for concrete, use \( f_{c'} = 30 \text{ MN/m}^2 \).

Solution
From Eq. (4.17),
\[
q_u = 1.3 c' N_e + q N_q + 0.4 \gamma B N_y
\]
\[
N_e = 5 \tan^6 \left( 45 + \frac{\phi'}{2} \right) = 5 \tan^6 \left( 45 + \frac{31}{2} \right) = 48.8
\]
\[
N_q = \tan^6 \left( 45 + \frac{\phi'}{2} \right) = \tan^6 \left( 45 + \frac{31}{2} \right) = 30.5
\]
\[
N_y = N_q + 1 = 30.5 + 1 = 31.5
\]

Hence,
\[
q_u = (1.3)(32 \times 10^3 \text{ kN/m}^2)(48.8) + (17 \times 2)(30.5) + (0.4)(25)(2.5)(31.5)
\]
\[
= 2030.08 \times 10^3 + 1.037 \times 10^3 + 0.788 \times 10^3
\]
\[
= 2031.9 \times 10^3 \text{ kN/m}^2 \approx 2032 \text{ MN/m}^2
\]
\[
q_{u(modified)} = q_u(\text{RQD})^2 = (2032)(0.5)^2 = 508 \text{ MN/m}^2
\]
\[
q_{ul} = \frac{508}{4} = 127 \text{ MN/m}^2
\]

Since 127 MN/m² is greater than \( f_{c'} \), use \( q_{ul} = 30 \text{ MN/m}^2 \).
The end