King Saud University

Department of Mathematics

Final Examination

ACTU 461 - Mathematics of Finance (2)

(11/4/1438 H, Time 3H)

(3 pages)

Exercise 1. [2+2.5+2.5]

The current price of XYZ stock is 45.34 per share. The risk-free effective rate of interest is 5% per annum. XYZ stock is expected to pay a dividend of 1.20 per share 6 months from now.

1) Find the fair price of a 9 month forward contract on this stock.
2) Suppose that the price of a 9 month forward contract on one share of XYZ stock is 43.56. Describe an arbitrage opportunity.
3) Suppose that the price of a 9 month forward contract on one share of XYZ stock is 47.56. Describe an arbitrage opportunity.

Exercise 2. [2.5 +2.5]

XYZ stock cost 55 per share. A three month forward contract on XYZ stock costs 58.

1) Calculate the annualized forward premium.
2) Calculate the 12-month forward price.

Exercise 3. [1+1+3]

1) Give the definition of a call option
2) Give the graph of the payoff of an European long call option with strike price K as a function of the spot price S_t at any time t.

3) Consider two European call options on a stock worth $S_0 = 32$ both with expiration date exactly one year from now and the same nominal amount. The risk-free annual rate of interest compounded continuously is 5%. One call option has a strike price 30 and the other one 35. We suppose that the price of the 30-strike call option is 7 and the price of 35-strike call option is 8. Describe an arbitrage opportunity and its minimum profit.

Exercise 4. [1+2+2]

1) Give the put-call parity formula in term of the forward price if we use an effective interest rate.
2) An investor is analyzing the cost of two year, European options for aluminum and zinc at a particular strike price and a risk-free annual effective interest rate of 6%.
   a) For each ton of aluminum, the two year forward price is 1400, a call option costs 700 and a put option costs 550.
      Calculate the strike price
   b) For each ton of zinc, the two year forward price is 1600 and a put option costs 550.
      Calculate the cost of a call option per ton of zinc.

Exercise 5. [6]   long

Stock XYZ has the following characteristics:

- The current price is 40 $= S_0$
- The price of a 35-strike 1-year European call option is 9.12 $e^{-0.08}(35,1) = 9.12$
- The price of a 40-strike 1-year European call option is 6.22
- The price of a 45-strike 1-year European call option is 4.08

The annual effective risk-free interest rate is 8%

Let S be the price of the stock one year from now.

Determine the range for S such that the 45-strike call produce a higher profit than the 40 strike call, but a lower profit than the 35-strike call.

(Hint. $40.44 < S < 42.31$)

Exercise 6. [6]

An investor inter a long position in a futures contract on an index (F) with a notional value of $200 \times F$ expiring in one year. The index pays an annual continuously dividend yield of 4%, and the continuously compounded risk-free interest rate is 2%.

At the time of purchase, the index price is 1100. Three months later, the investor has sustained a loss of 100. Assume the margin account earns an interest rate of 0%.

Let S be the price of the index at the end of month three. Calculate S

Exercise 7 [3+3].

Suppose that the LIBOR discount factors $P(0, t_j)$ are given in the table below. Consider a 2-year swap with quarter payments whose floating payment are found using the LIBOR rate compiled three months before the payment is made. The notional amount of the swap is 10000.

LIBOR discount rates $P(0, t_j)$	0.97	0.94	0.92	0.89	0.86	0.84	0.83	0.79
Time (months)	3	6	9	12	15	18	21	24

1) Calculate the par-swap rate.
2) Calculate the net payment made in this contract in 15 month if the quarter LIBOR interest rate compiled in 12 months is 2.3%.
   Who will pay this payment, the fixed rate side or the floating rate side?