جامعة الملك سعود
King Saud University

College of Science.
Department of Statistics & Operations Research

كلية العلوم
قسم الإحصاء وبحوث العمليات

First Midterm Exam
Academic Year 1444-2023-Third Semester

Exam Information معلومات الامتحان			
Course name	Modeling and Simulation	النمذجة والمحاكاة	اسم المقرر
Course Code	OPER 441	441 بحث	رمز المقرر
Exam Date	18-5-2023	28-10-1444	تاريخ الامتحان
Exam Time	09:00 am		وقت الامتحان
Exam Duration	2.5 hours	ساعتان ونصف	مدة الامتحان
Classroom No.			رقم قاعة الاختبار
Instructor Name			اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of ☐ PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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تعليمات عامة:

- عدد صفحات الامتحان ☐ صفحة. (باستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	Understanding the processes and steps for building a simulation model			
2	Implement an inverse cumulative distribution function based random variate generation algorithm			
3	Explain and implement the convolution algorithm for random variate generation			
4	Explain and implement the acceptance rejection algorithm for random variate generation			
5	Compute statistical quantities from simulation output			
6	Generate random numbers from any given distribution discrete or continuous			
7	Building simulation models from basic applications			
8				

EXAM COVER PAGE

Question #1: Answer the following with *True* or *False*: 15 pt

F	***False***	1. A Random experiment is any experiment whose outcome is known in advance for certain.
T	***True***	2. Sample Space of the experiment is all possible outcomes of the random experiment
T	***True***	3. The elements of the sample space of any experiment must be always mutually exclusive
F	***False***	4. the experiment of the flipping of a coin one time has a sample space of four possible outcomes.
F	***False***	5. Events are subset of the sample space and must be always mutually exclusive
T	***True***	6. The probability of the sample space is always equal to 1.
F	***False***	7. The probability distribution for any random experiment (S) must satisfy: for all $E \in S$ then $0 \leq P(E) \leq 1$ ***and P(S)=1***
F	***False***	8. In the coin tossing example, if we assume that a head is equally likely to appear as a tail, then we would have P(H)=P(T) = 0.45
F	***False***	9. if we had a biased coin with head three times as likely to appear as a tail, then we would have P(H)=0.6 and P(T) = 0.2
T	***True***	10. For any event E, the event E didn't happen is E^c
T	***True***	11. A and B are independent if and only if P{AB} = P{B}P{A}
T	***True***	12. Simulation model can be used to evaluate different alternatives and give an optimal solution.
F	***False***	13. The sample space in a random experiment is always determined and unique to everyone.
T	***True***	14. Simulation modeling is not good if there is less data or no estimates available.
F	***False***	15. In call center model with two lines it is impossible to lose any incoming call.
F	***False***	16. In Bank simulation, the variable (X = number of kids with a customer) is a state variable for the system.
T	***True***	17. The measures of simulation change every time a new run of simulation is performed
F	***False***	18. Every simulation run for the same model give the same results.
T	***True***	19. The uniform distribution is used when all values have the same chance to appear.
T	***True***	20. If the random variable has mean value equals to the variance then it must have a Poisson distribution.
T	***True***	21. The normal distribution always has a bill shape around the mean
T	***True***	22. The sequence of random numbers generated from a given seed is called a random number a *Stream.*

F	***False***	23. LCG has full period if and only if we get exactly (m–1) random numbers. ***(different random numbers and with the same seed)***
F	***False***	24. Always, if the LCG repeat the starting value R_0 then the function has a full period.
F	***False***	25. Every LCG must satisfy all three conditions to have full period
F	***False***	26. Every time you run the simulation model you get the same output data.
F	***False***	27. The LCG is the function, it is possible always to find values for: R_0 , a , c and m to generate more than m different random numbers.
F	***False***	28. The LCG is the function $X_n = (aX_{n-1} + c) /(m)$.
T	***True***	29. The LCG is always used to generate pseudo-random numbers between 0 and 1.
F	***False***	30. The number of trials until 1st two success is a binomial distribution.
F	***False***	31. The Uniform distribution has a single mode value.
T	***True***	32. The normal distribution always has the mean equals to the median
F	***False***	33. The Gamma distribution always have the memory less property
F	***False***	34. If the distribution has a function the CDF then it must have an inverse transform.
F	***False***	35. The Exponential distribution has the CDF function $\lambda e^{-\lambda x}$,
F	***False***	36. The inverse transform takes random values from Uniform (a,b) and transform it to random values from a given distribution.
F	***False***	37. The Gamma distribution is a special case from the Exponential distribution.
T	***True***	38. The Geometric distribution is a special case from the Binomial distribution.

Question #2: 10 pt

A traffic control engineer reports that 75% of the cars passing through a check point are from Riyadh city. If at this check point, five cars are selected at random. Find:

1. The probability that four of them are from Riyadh city?
2. The probability that at least four of them are from Riyadh city?
3. The probability more than four cars will pass until one car from Riyadh city passes?
4. The expected number of cars that are from Riyadh city?
5. If you decided to simulate this system to approximate the above probabilities, what are the data that you will collect from the simulation model?

Solution

Pr{ cars passing through a check point from Riyadh city} = 0.75 = prob. Of success
The random variable X = number of cars passing through a check point from Riyadh city
Total number of cars selected = 5 cars = number of trials
Then X ~ Bionomial(p=0.75, n=5)

$$\Pr\{X = k\} = \binom{5}{k}(0.75)^k(0.25)^{5-k}$$

1. Pr{four of them are from Riyadh}

$$\Pr\{X = 4\} = \binom{5}{4}(0.75)^4(0.25)^{5-1} = 5(0.75)^4(0.25) \quad = 0.396$$

2. Pr{ at least 4 cars from Riyadh} = P{X=4} + P{X=5}

$$= 0.396 + \binom{5}{5}(0.75)^5(0.25)^0 = 0.396 + 0.2373 = 0.6328$$

3.P{more than four cars pass until one car from Riyadh city passes}

= 1- P{less or eq. than four cars pass until one car from Riyadh city passes} = 1-P{1 car} –

= 1-P{1 car} - P{2 cars until Riyad pass} - P{3 cars until Riyad pass}

= 1- [(0.75) + (0.25)(0.75) + $(0.25)^2$ (0.75) + $(0.25)^3$ (0.75)] = 1 – [0.99609375] = 0.00390625

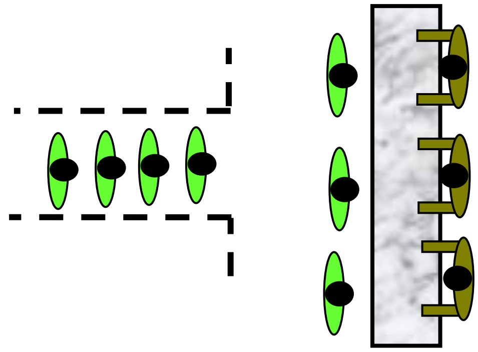
4.E[number of cars from Riyadh] = (5)(0.75) = 3.75 cars

5. To simulate this system we need the following data:

- The order of the car
- The time of arrival of the car
- the origin of the car (Riyadh, Not Riyadh)

Question #3: 10 pt

A service station has three servers and a single waiting line. The servers serve customers in the order in which they arrive. Servers may leave the service at any time for taking break. Also, customers may leave the waiting queue without service due to long waiting time. The service time of the customers changes according to their gender and the type of service they request. The service facility provide four types of services.



In the list below at least 10 **state variable** and 5 **attributes** and define the possible values of each one?

	Definition of Variable	*Type*	*Values*
1	*Number of busy servers at time (t)*	*State Variable*	*0,1,2,3*
2	*Number of customers in the service station at time (t)*	*State Variable*	*0,1,2, 3, …*
3	*Number of customers entered the service station at time (t)*	*State Variable*	*0,1,2,3,…*
4	*Number of customers departed the station with service at time (t)*	*State Variable*	*0,1,2,3,… ,number entered*
5	*Number of customers departed the waiting line without service at time (t)*	*State Variable*	*0,1,2,3,… ,number waiting*
6	*Number of customers waiting for service at time (t)*	*State Variable*	*0,1,2,3,…*
7	*Number of Idle servers at time (t)*	*State Variable*	*0,1,2,3*
8	*Number of customers in the service at time (t)*	*State Variable*	*0,1,2,3*
9	*System is empty at time (t)*	*State Variable*	*0,1*
10	*Number of servers taking a break at time (t)*	*State Variable*	*0,1,2,3*
11	*The time until customer depart the system without service*	*Attribute Variable*	*PT>0*
12	*Service time of the customer*	*Attribute Variable*	*ST>0*
13	*Arrival time of the customer*	*Attribute Variable*	*AT>0*
14	*The gender of the customer*	*Attribute Variable*	*M,F*
15	*Break time of the Server*	*Attribute Variable*	*BT>0*

Question #4:

The length of time for one customer to be served at a bank is a random variable X that follows the exponential distribution with a mean of 4 minutes.

1. What is the probability that a customer will be served in less than 2 minutes?
2. Given that the customer spends more than 5 minutes until now, what is the probability that a he will need more than 4 minutes to complete his service?
3. Given that the bank now has more than 100 customers, what is the expected number of customers completed their service in one hour from now?
4. Draw the flowchart that will simulate this system to approximate the above probabilities? (**Use the command *Generate RV from Dist.*--- to complete your flowchart**)

Solution

The mean of customer service in the bank E[X] = 4 min

The rate of service = λ = 1/E[X] = ¼ customer/min = 15 Cut./hr.

Then X ~ Exponential (λ = 0.25)

$$\Pr\{X = t\} = \frac{1}{4}\, e^{-\frac{t}{4}}$$

1. Pr{ customer will be served in less than 2 minutes } $= \Pr\{X \leq 2\ min\}$

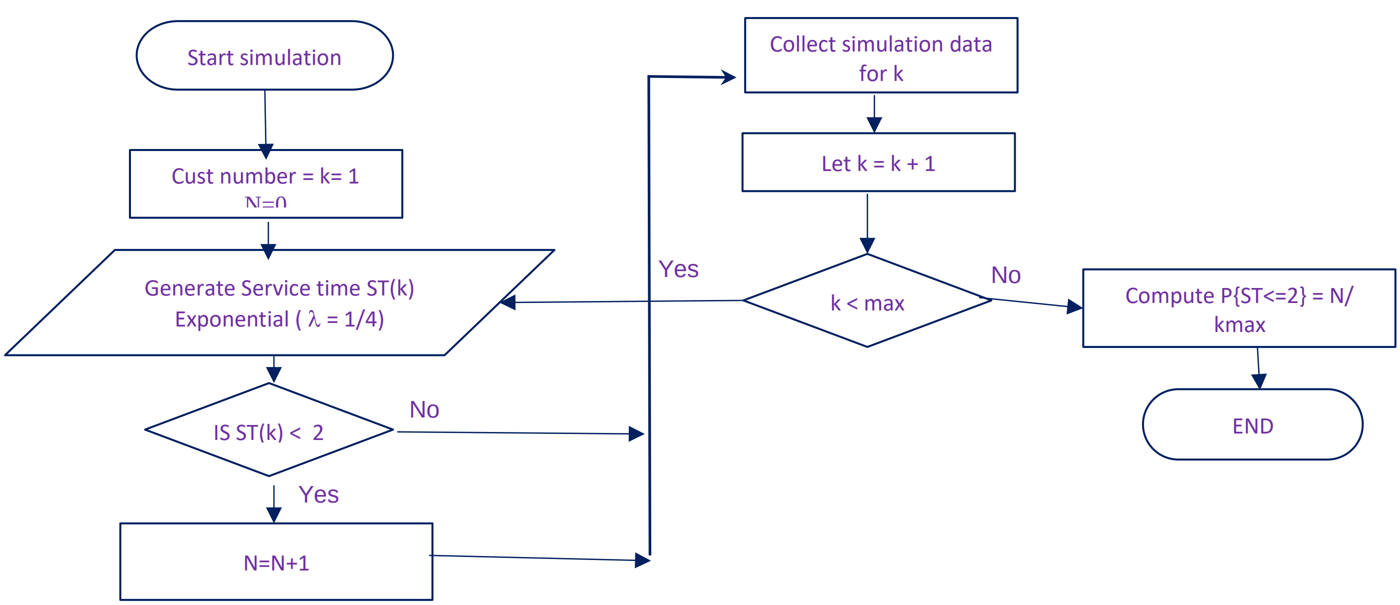
$$\Pr\{X \leq 2\ min\} = 1 - e^{-\frac{2}{4}} = 0.393$$

2. Pr{ X ≥9 | X≥5} = Pr{X ≥4} = $e^{-\frac{4}{4}} = 0.368$

3. E[number of customers served in on hour] = 15 customers per hour

4. To simulate this system we need the following data:

The order of the customer, The time of customer service



Question #5:

Consider the following LCG generator m=64, a=5, c=3

Answer the following:

a) Using the three conditions, prove that this LCG has a full period.

b) Generate the first 10 random numbers between 0 and 1 from the above function starting from $R_0 = 2$.

c) Consider an investment portfolio that changes randomly with percentage x% according to the following pdf:

$$f(x) = \begin{cases} \frac{4}{7}x^3 ; & -1 \leq x \leq 2 \\ 0 ; & otherwise \end{cases}$$

For each level (x%), the portfolio remains a random amount of time following an exponential distribution with average 2 days. Using the above LCG, simulate this process for with $R_0 = 2$ stream for x% and $R_0 = 13$ stream for time. Starting with x = 0. End the simulation at the 10th change. Estimate the average change at the end of simulation.

n	R(n)	R(n+1)	U(n+1)
1	Seed= 0	1	0.053
2	1	6	0.316
3	6	12	0.632
4	12	4	0.211
5	4	2	0.105
6	2	11	0.579
7	11	18	0.947
8	18	15	0.789
9	15	0	0.000
10	Seed= 3	16	0.842
11	16	5	0.263
12	5	7	0.368
13	7	17	0.895
14	17	10	0.526
15	10	13	0.684
16	13	9	0.474
17	9	8	0.421
18	8	3	0.158
19	Seed= 14	14	0.737

Part a)

Cond#1: divisors of $m \in \{1,2,4,8,16,32,64\}$

Divisors of $c \in \{1,3\}$

The only common divisor between m and $c = 1$

Cond#1 is TRUE

Cond#2: prime divisors of $m \in \{1, 2\}$

all must divide $4 = a-1$

4/1 = 4 and 4/19 = 0.2105 ➔ Cond#2 is not met

Then LCG doesn't have a full period.

Part b)

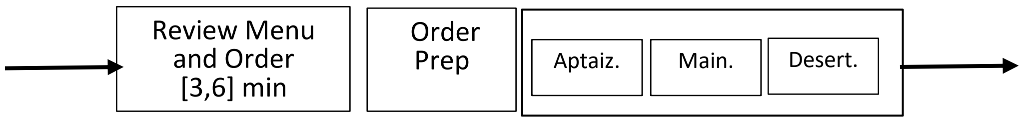
From the table: Number of streams = 3 streams

Part c)

See last column in the table

Question #6:

Customers arrive to a restaurant according to a random process follows the Poisson process with rate 15 customers per hour. The customers take their seats on one of the tables available, choose their entries, waits for their order to be presented and they start eating. The processes are illustrated as follows:



- The Review of the Menu and placing an order takes a random time between 3 to 6 min.
- The preparation of the order takes a random time follows the exponential distribution with mean equals to three times the time of placing the order.
- Eating time is divided into three parts:
    a) Appetizers time: follows exponential with mean 5 min.
    b) Main course time: follows exponential with mean 15 min.
    c) Appetizers time: follows exponential with mean 10 min.

Answer the following:

1. Define all random processes involved in the system.
2. Build the algorithm for simulating the table occupation in the restaurant and draw the flow chart for it.
3. Simulate the system for ten customers to estimate average number of tables occupied per hour.

Question #7:

Consider a Car dealer who has a show room that can take up to 10 cars. The manager review the sales and demand on cars by the end of each week. The manager doesn't want the cars in the showroom go below 5 cars by the end of the week, therefore, if he observed number of cars in the showroom is ($y<5$) at end of the week then the manager will order enough new cars to exactly fill the showroom to the maximum. The new cars will be delivered immediately next day. Also, If the manager observed storage level is at end of the week is $y \geq 5$, then no new cars will be ordered. As to satisfy customers, the manager can accept no more than one car only in shortage demand

Answer the following:

1. Write the Simulation algorithm and the flow chart of this system.
2. Use the following input data to simulate the system
3. Estimate expected number of cars in showroom per week from simulation.
4. Estimate probability of that the show room has less than 3 cars by the end of the week from simulation.

wk	Car Demand	Start Storage	End Storage	Make order (Y/N)	Order Quantity	Lost demand
1	3	10	7	N	0	0
2	2	7	5	N	0	0
3	8	5	-1	Y	11	2
4	2	10	8	N	0	0
5	4	8	4	Y	6	0
6	4	10	6	N	0	0
7	8	6	-1	Y	11	1
8	5	10	5	N	0	0
9	2	5	3	Y	7	0
10	1	10	9	N	0	0
11	6	9	3	Y	7	0
12	3	10	7	N	0	0
13	10	7	-1	Y	11	2
14	3	10	7	N	0	0
15	2	7	5	N	0	0