Work Sampling

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Work Sampling

2. Statistical Basis of Work Sampling

Statistical Basis of Work Sampling

- Statistical Basis of work sampling:
  - **Binomial distribution**:
  - parameter p = true proportion of time spent in a given category of activity
- There are usu. multiple activity categories (e.g. 1)
  - we have $p_1, p_2, \ldots, p_k, \ldots, p_K$ proportions
  - for K different activity categories
  - sum of proportions must equal unity

Statistical Basis of Work Sampling

- Binomial distribution can be approximated by **normal distribution** (for computational convenience), where

$$\mu = np$$

$$\sigma = \sqrt{np(1-p)}$$

  - μ: mean
  - n: total number of observations
  - σ: standard deviation
  - p: proportion
- Normal approximation:
  - quite sufficient due to large number of observations in typical WS study

Alternative Parameters

- μ and σ are converted back to proportions:
  - dividing by number of observations:

$$p = \frac{\mu}{n} = \frac{np}{n}$$

$$\sigma_p = \frac{\sqrt{np(1-p)}}{n} = \sqrt{\frac{p(1-p)}{n}}$$

  - σ_p: standard deviation of proportion p

Estimating the Proportion *p*

- In a sampling study:
  - $\hat{p}$: proportion of total number of observations devoted to an activity category of interest
  - $\hat{p}$: is estimate of true value of population proportion p
  - for $\hat{p}$ to be good estimate, it must:
    - **Absent of bias**
      - e.g. bias when human subjects know they are being observed in WS study
      - Solution: randomize observation times
    - **Low variance** (variance $= {\sigma_p}^2$):
      - Increasing number of observations (n)*

2.1 Confidence Intervals

- Due to statistical error,
  - $\hat{p}$ is not exactly equal to p
  - estimating how close $\hat{p}$ is to p depends on:
    - defined error range
    - confidence level

2.1 Confidence Intervals

- Confidence interval for std. normal distribution relative to p expressed as follows (fig. 1):

$$Pr\left(-z_{\alpha/2} < \frac{\hat{p} - p}{\hat{\sigma}_p} < +z_{\alpha/2}\right) = 1 - \alpha$$

$\hat{\sigma}_p$: calculated value of proportion std. dev.

- Rearranging (fig. 2):

$$Pr\left(\hat{p} - z_{\alpha/2}\hat{\sigma}_p < p < \hat{p} + z_{\alpha/2}\hat{\sigma}_p\right) = 1 - \alpha$$

- In words:
  - Probability that actual value of p lies between $\hat{p} - z_{\alpha/2}\hat{\sigma}_p$ and $\hat{p} + z_{\alpha/2}\hat{\sigma}_p$ is $(1 - \alpha)$
  - Or: value of p lies between $\hat{p} - z_{\alpha/2}\hat{\sigma}_p$ and $\hat{p} + z_{\alpha/2}\hat{\sigma}_p$ at a confidence level of $(1 - \alpha)$

2.1 Confidence Intervals

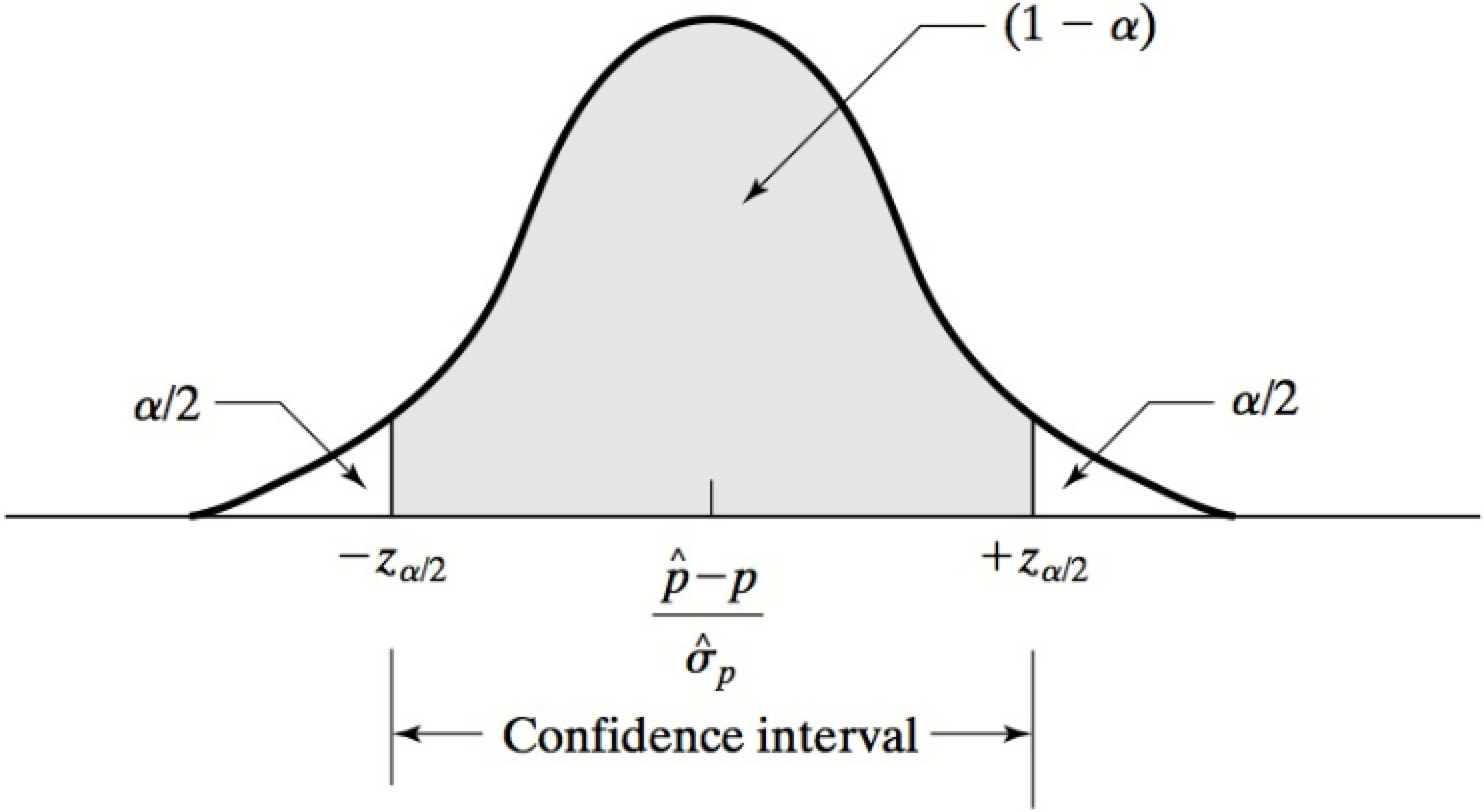

Figure 1 Definition of a confidence interval in the standard normal distribution.

2.1 Confidence Intervals

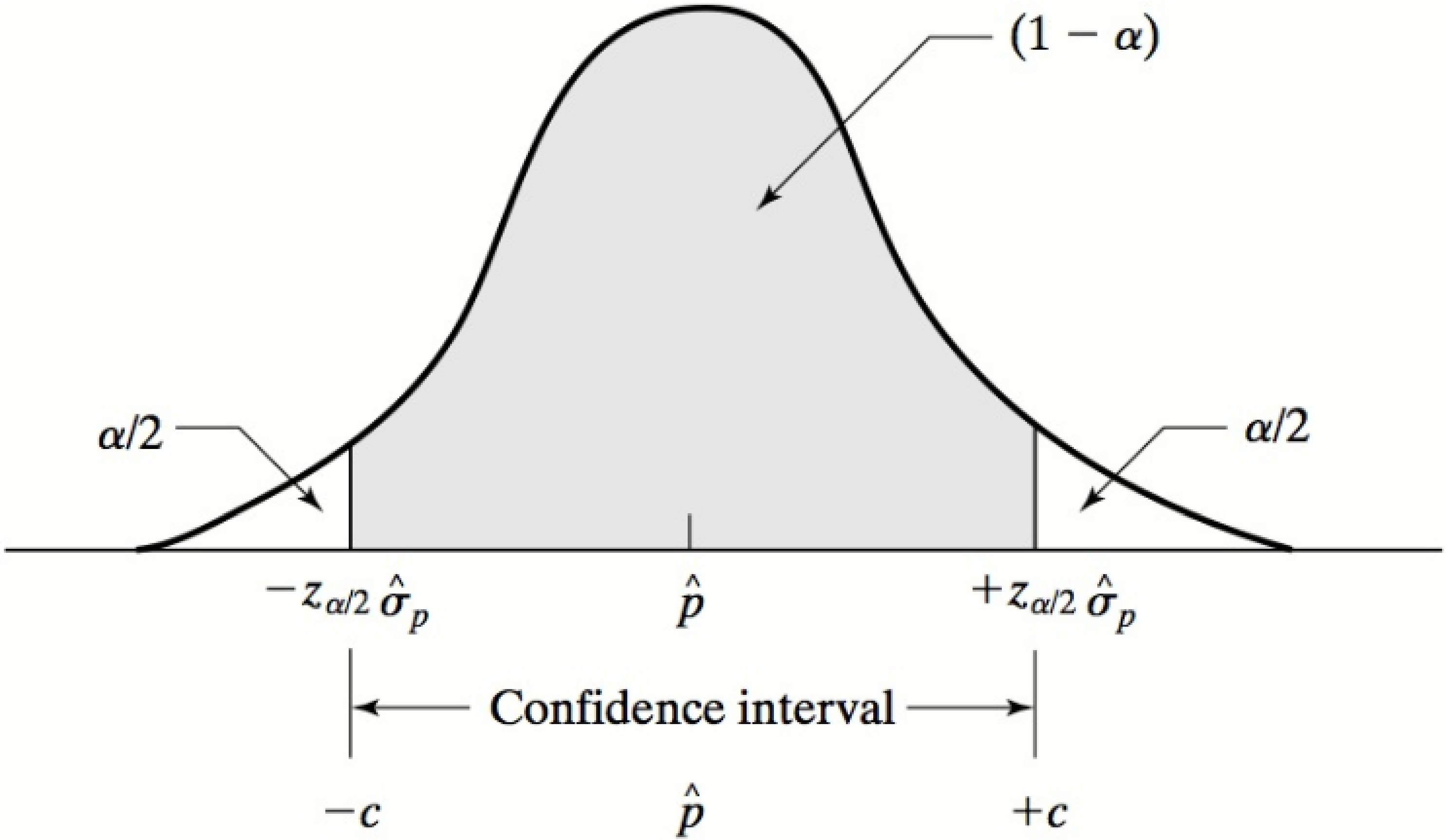

Figure 2 Confidence interval superimposed on the normal distribution, with $(1-\alpha)$ and $Z_{\alpha/2}$ defined.

2.1 Confidence Intervals

- Confidence levels used in work sampling
  - Typically: 90%, in which case $z_{\alpha/2} = 1.65$
  - Also: 95%, in which case $z_{\alpha/2} = 1.96$
  - Other confidence levels and $z_{\alpha/2}$ values found in tables of std. normal distribution

Example 2: Confidence Interval

- Determine the 95% confidence interval for the proportion of time spent setting up the machines, category (1), in Example 1.

Remember (example 1):

- A total of 500 observations were taken at random times during a one-week period (40 hours) on 10 machines.

Category	Proportion	Hrs per category
(1) Being set up	75/500 = 0.15	0.15 x 40 = 6
(2) Running production	300/500 = 0.60	0.60 x 40 = 24
(3) Machine idle	125/500 = 0.25	0.25 x 40 = 10
	1.00	40

Example 2: Solution

Solution: From the solution of Example 1, we know that the proportion of time spent setting up machines is $\hat{p} = 0.15$. Computing the standard deviation using equation (4), given that there are a total of 500 observations ($n = 500$),

$$\hat{\sigma}_p = \sqrt{\frac{0.15(1 - 0.15)}{500}} = 0.01597$$

For the 95% confidence level, the corresponding $z_{\alpha/2} = 1.96$; thus

$$\hat{p} - z_{\alpha/2}\,\hat{\sigma}_p = 0.15 - 1.96(0.01597) = 0.15 - 0.01313 = 0.1187$$

and

$$\hat{p} + z_{\alpha/2}\,\hat{\sigma}_p = 015 + 1.96(0.01597) = 0.15 + 0.013 = 0.1813$$

Accordingly, we can state that at the 95% confidence level, the true value of the setup time proportion lies between 0.1187 and 0.1813. ■

2.2 Number of Observations Required

- How can statistical errors in work sampling be reduced?
  - Increasing the number of observations (n)
  - Thus,
    - Accuracy/precision of $\hat{p}$ (estimate of given proportion of interest p) is increased
    - Limits of the confidence interval around it can be narrowed*
  - However, increasing $n \Rightarrow$ more time/costs
    - $\Rightarrow$ balance bet. cost/time and accuracy):
    - determine n required to **achieve a given confidence interval** about estimate of p

2.2 Number of Observations Required

Step 1: define **confidence interval**

- We need to decide on two parameters:
  1. **Confidence level**, $1-\alpha$
     - This allows us to find corresponding $z_{\alpha/2}$
  2. **Half-width** of the confidence interval, c
     - it's desired acceptable deviation from p
     - think of c as tolerance around p

- $\Rightarrow$ confidence interval $= p \pm c$ (fig. 2)
  - Note, c corresponds to $z_{\alpha/2}\hat{\sigma}_p$
  - $\Rightarrow \hat{\sigma}_p = \dfrac{c}{z_{\alpha/2}}$

2.2 Number of Observations Required

Step 2: find $\boldsymbol{n}$ to achieve specified confidence level:

- Noting equation:

$$\hat{\sigma}_p = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

- We rearrange to get $n = f\left(\hat{p}, \hat{\sigma}_p\right)$:

$$n = \frac{\hat{p}(1-\hat{p})}{\hat{\sigma}_p{}^2}$$

  - Where $\hat{\sigma}_p$ (last slide): $\hat{\sigma}_p = \frac{c}{z_{\alpha/2}} \Rightarrow$

$$n = \frac{\hat{p}(1-\hat{p})}{\hat{\sigma}_p{}^2} = \frac{\left(\boldsymbol{z_{\alpha/2}}\right)^{\boldsymbol{2}} \widehat{\boldsymbol{p}}(\boldsymbol{1}-\widehat{\boldsymbol{p}})}{\boldsymbol{c^2}}$$

e.g. 3 Finding Required Observations

In the context of our previous examples, determine how many observations will be required to estimate* the proportion of time used to set up the 10 machines in the automatic lathe section. The confidence interval must be within ± 0.03 of the true proportion, which the foreman initially estimates to be $\hat{p} = 0.20$. A 95% confidence level will be used.

Example 3: Solution

Solution: At the 95% confidence level, $z_{\alpha/2} = 1.96$. The confidence interval is 0.20 ± 0.03, so the half-width of the interval c is 0.03. From Equation (8),

$$\hat{\sigma}_p = 0.03/1.96 = 0.0153$$

Using this value in equation (9), we obtain the required number of observations:

$$n = \frac{0.20(1-0.20)}{(0.0153)^2} = 683.5, \text{ rounded up to 684 observations.}$$ ■

- Note how if $\hat{p}$ is different than $p \Rightarrow$ recalculate n
  - If computed n > original $n \Rightarrow$ **additional observations** must be taken to maintain desired confidence level (here: $684 - 500$)
  - If computed n < original $n \Rightarrow$
    - confidence level > specified level,
    - or: confidence level < original

2.3 Determining Average Task Times

- Time standards established by work sampling:
  - not as accurate as those set by other methods
  - not appropriate for wage incentive plans
  - can be used when other work measurement techniques are not practical
    - e.g. when length of time required to use other techniques would be excessive
- We will show how to use WS to determine:
  - Average task time
  - Normal time per work unit
  - Standard time per work unit

2.3 Determining Average Task Times

- Average task time:
  - how much **time** required to complete **average work unit**
  - no consideration of worker's performance during work
- Time standard by work sampling
  - requires observer to correctly **classify activity category** during observation, and
  - also **rating worker's performance**
- In both cases:
  - quantity of work units completed during duration of WS study must be counted

2.3 Determining Average Task Times

- Average task time for a given work category:
  - computing total time for category
  - then dividing by total count of work units produced by that category:

$$T_{ci} = \frac{p_i(TT)}{Q_i}$$

  - T_{ci}: **avg. task time** (or $c$ycle time for task i)
  - p_i: proportion of observations associated with category
  - TT: total time of WS study (note, all time is in hr or min)
  - Q_i: total quantity associated with category i (during TT)

Example 4 Average Task Times in WS

Suppose in Example 1 that a total of 1572 work units were completed by the 10 machines and that a total of 23 setups were accomplished during the 5-day period. Determine,

(a) the average task time per work unit during production, and

(b) the average setup time

Example 4 Solution

Solution: Let us first compute the total time of the work sampling study. The duration of the study was 40 hr, but there were 10 machines (subjects), so the total time is 40 hr × 10 machines:

$$TT = (40 \text{ hr})(10 \text{ machines}) = 400 \text{ hr}$$

(a) The 1572 work units are associated with the "running production" category, whose proportion $\hat{p} = 0.60$.

$$T_c = \frac{0.60(400)}{1572} = 0.1527 \text{ hr} = 9.16 \text{ min}$$

(b) The 23 setups are associated with "being set up," whose proportion $\hat{p} = 0.15$.

$$T_{su} = \frac{0.15(400)}{23} = 2.609 \text{ hr} = 156.52 \text{ min/setup}$$

■

Example 4 Notes

- Computed times in this e.g.: of limited value:
  - unlikely that 1572 work units are all the same
  - 23 setups performed during observation:
    - 23 different batches $\Rightarrow$ different parts design
    - $\Rightarrow$ different machining times*
    - $\Rightarrow$ also different setup times
- However, e.g. provides useful information:
  - avg. T_{su} per batch: $156.52\,min = 2.609\,hr$
  - the avg. T_c per part: $9.16\,min$
  - $\Rightarrow$ foreman may make a judgment that $2.609\,hr$ is longer than required for setup

2.3 Determining Standard Times

- Cases to use WS to find time standards
  - indirect labor activities
  - clerical office work
  - in general: inherent variability in tasks
    - e.g. repair work on digital cameras
    - e.g. products returned to the factory
- Conditions for using WS to find T_{std}
  - homogeneity in work units (e.g. digital cameras) for which standard is set
  - rate performance of worker during each observation*
  - identifying the category of activity

2.3 Determining Standard Times

- First determine **normal time** for activity i

$$T_{ni} = \frac{p_i(TT)(\overline{PR}_i)}{Q_i}$$

  - T_{ni}: $n$ormal time for work unit in work category i, min
  - $\overline{PR}_i$: average value of performance ratings for all observations in category i

- Then determine **standard time**

$$T_{stdi} = T_{ni}\left(1 + A_{pfd}\right)$$

  - T_{stdi}: standard time for work unit i, min
  - A_{pfd}: allowance (personal, fatigue, delays)