College of Science.
Department of Statistics & Operations Research

كلية العلوم
قسم الإحصاء وبحوث العمليات

Second Midterm Exam
Academic Year 1443-1444 Hijri- First Semester

Exam Information معلومات الامتحان			
Course name	**Modeling and Simulation**	التمذجة والمحاكاة	اسم المقرر
Course Code	**OPER 441**	441 بحث	رمز المقرر
Exam Date	**29-11-2023**	7-7-1444	تاريخ الامتحان
Exam Time	1:30 pm to 3:30 pm		وقت الامتحان
Exam Duration	**2.0 hours**	ساعتان	مدة الامتحان
Classroom No.			رقم قاعة الاختيار
Instructor Name			اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

تعليمات عامة:

- Your Exam consists of [   ] PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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- عدد صفحات الامتحان [   ] صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة
This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	To know the basics of pseudo random generation and apply different methods of random generation techniques			
2	Chose and fit theoretical distribution on collected data			
3	Define and compute performance measures from simulation models			
4	Recognize and analyze simple models and its main elements for simulation			
5	Understanding how to use computer software (ECXEL) for simulation models			
6	use appropriate statistical techniques to analyze and evaluate outputs of simulation models			
7	Generate random variates from different probability functions and directly from collected data			
8	build simple simulation models of real-life problems			

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Question #1: (15 point)

A continuous random variable (X) ranges from −3 to 4 is defined by the following CDF:

$$F(x) = \begin{cases} 0, & x \leq -3 \\ \dfrac{1}{2} + \dfrac{x}{6}, & -3 < x \leq 0 \\ \dfrac{1}{2} + \dfrac{x^2}{32}, & 0 < x \leq 4 \\ 1, & x > 4 \end{cases}$$

(1) Write the inverse transform for this random variable.

(2) Let (X) be the percentage of change in a week for a given share. Given that the closing price of this week for this share is 123 SR write the closing price for this share for the next 8 weeks. (use the table below)

(3) From the simulated date, what is the probability of closing with increase.

Week #	1	2	3	4	5	6	7	8	
U1	0.353	0.034	0.672	0.622	0.408	0.218	0.889	0.643	
U2	0.962	0.981	0.781	0.313	0.600	0.910	0.808	0.632	
%-change	-0.88	-2.80	2.35	1.98	-0.55	-1.69	3.53	2.14	
Closing Price	121.92	119.56	125.89	125.43	122.32	120.92	127.34	125.63	

(1) Inverse transform: let R~U[0,1]

$$X = \begin{cases} 6(R - 1/2) & 0 \leq R \leq 1/2 \\ \sqrt{32(R - 1/2)} & 1/2 \leq R \leq 1 \end{cases}$$

(2) Closing Price: Let X = be the percentage of change in a week for a given share
Closing Price of week (n) is CP(n) Closing Price of week (n)
CP(n) = CP(n) + X CP(n-1)/100

(3) probability of closing with increase = 4/8=1/2

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Question #2: (15 point)

Consider a continuous random variable X with the following pdf

$$f(x) = \begin{cases} e^{2x}, & -\infty < x \le 0 \\ e^{-2x}, & 0 < x < \infty \end{cases}$$

(1) Derive the inverse transform to generate random values for X and apply it for Table(1)
(2) Use acceptance/rejection method to generate random values for X and apply it for Table(2) assuming that $x \in [-7,7]$

Table (1)

Week #	1	2	3	4	5	6	7	8	9	10
U1	0.570	0.462	0.055	0.571	0.062	0.760	0.701	0.493	0.082	0.261
U2	0.826	0.127	0.318	0.106	0.850	0.830	0.714	0.429	0.079	0.816
X	0.075	-0.040	-1.104	0.077	-1.044	0.367	0.257	-0.007	-0.904	-0.325

Table (2)

Week #	1	2	3	4	5	6	7	8	9	10
U1	0.57	0.462	0.055	0.571	0.062	0.76	0.701	0.493	0.082	0.261
U2	0.826	0.127	0.318	0.106	0.85	0.83	0.714	0.429	0.079	0.816
W	0.98	-0.532	-6.23	0.994	-6.132	3.64	2.814	-0.098	-5.852	-3.346
f(W)	0.141	0.345	0.000	0.137	0.0000	0.001	0.004	0.822	0.000	0.001
A/R	R	A	R	A	R	R	R	A	R	R
X	Repeat	-0.532	Repeat	0.994	Repeat	Repeat	Repeat	-0.098	Repeat	Repeat

Step 1. CDF:

$$if \ x \le 0 \rightarrow \quad F(x) = \int_{-\infty}^{x} e^{-2y} dy = \frac{1}{2} e^{2x}$$

$$if \ x > 0 \rightarrow \quad F(x) = \frac{1}{2} + \int_{-\infty}^{x} e^{2y} dy = \frac{1}{2}(1 - e^{-2x})$$

Step 2. Inverse CDF:

$$if \ x \le 0 \rightarrow \quad u = \frac{1}{2} e^{2x} \rightarrow \quad X = \frac{1}{2} \ln(2u) \quad with \ 0 \le u \le 0.5$$

$$if \ x > 0 \rightarrow \quad u = \frac{1}{2}(1 - e^{-2x}) \rightarrow \quad X = -\frac{1}{2} \ln(2 - 2u) \quad with \ 0.5 \le u \le 1$$

Acceptance/Rejection for $X \in [-7,7]$
g(x) = max f(x) = 1
c= 14
W(x) = g(x)/ 14 → W= -7 + 14 u

Algorithm

1. Generate u_1
2. compute W= -7 + 14 u_1
3. evaluate f(W) and g(W) = 1
4. choose u_2 and test if f(W) > u_2 → Accept W~f
5. Else repeat (1)

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Question #3: (15 point)

A machine is taken out of production (turn off the machine) either if it fails or after 5 hours shift of continuous operation, whichever comes first. From past data, it has found that the machine can operate without failure for a random amount of time (X) following a Weibull distribution with parameters α = 0.75 and β = 8:

$$f(x) = \frac{\alpha}{\beta^\alpha} x^{\alpha-1} e^{-(x/\beta)^\alpha}$$

Thus, the time until the machine is taken out of production can be represented as Y = min(X, 5).
   (1) Write a step-by-step procedure for generating Y.
   (2) Use your answer in (1) to apply on the following uniform numbers.
   (3) What is the probability that the machine operates the shift without failure.
   (4) What is the average machine operation time per shift.

Week #	1	2	3	4	5	6	7	8	9	10
U1	0.201	0.417	0.797	0.316	0.042	0.190	0.697	0.083	0.867	0.354
X	1.091	3.514	14.903	2.200	0.120	1.003	10.134	0.307	20.393	2.653
Y	1.091	3.514	5	2.200	0.120	1.003	5	0.307	5	2.653

Inverse X = 8 (-ln(1-u))$^{4/3}$

Algorithm:

1. Get a uniform u~U[0,1]
2. Compute X = 8 (-ln(1-u))$^{4/3}$
3. If X <=5 → Y = X
4. Else if X>= 5 → Y =5

probability that the machine operates the shift without failure = 3/10 = 0.3

average machine operation time per shift = ΣY/10 = 25.887/10 = 2.5 hours

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Question #4: (15 point)

Students arrive at a self-service cafeteria at the rate of one every **30±20 [10,50] seconds**. It is estimated that **40% of students go to the sandwich bar**, where every student prepares has own **sandwich in 60±30 [30,90] seconds**. The rest go to the main counter, where one server spoons the prepared **meal onto a plate in 45±30 [15,75] seconds**. All students take their seats in the cafeteria and spend **20 ± 10 [10,30] minutes** eating. After eating, **10% of the students go back for dessert**, and return to their table to spend an **additional 10±2 [8,12] minutes** in the cafeteria.

1. Simulate until 10 people have left the cafeteria using the following table of U[0,1] numbers.
2. At the final simulation time, estimate the following from the simulation data:
   a. How many students are still in the cafeteria
   b. What percentage of students at the sandwich bar.
   c. What percentage of students at the main course counter.
   d. What percentage of students on tables
   e. What percentage of students take dessert.
3. From the simulation data, what is the average time that a student who gets a sandwich spends in the cafeteria until he leaves after finishing his entire meal.

1. RP#1: Time between arrivals ~[10,50] seconds → TB(i) = 10+[41u] , AT(i) = AT(i-1) +TB(i)

2. RP#2: Choose sandwich SB/ Sandwich/Main Course MC: if u<0.4 → SB Else MC

3. RP#3: Sandwich Time SBT(n) ~ [30,90] seconds. → TB(i) = 30+[61u] seconds

4. RP#4: Main Course Time MCT(n) ~[15,75] → MCT(i) = 15+[61u] seconds

5. RP#5: Eating Time ET(n)~ [10,30] minutes ET(n)= 10+[21u] minutes

6. RP#6: Desert YD/ND: if u<0.1 → YD Else ND

7. RP#7: Eating Desert Time EDT(n)~ [8,12] minutes : EDT(n)= 8+[5u] minutes

	Arrival Time AT(n)		Sandwich/Main Course SB/MC	Sandwich Time SBT(n)	Main Course Time MCT(n)	Eating Time ET(n)	Desert YD/ND	Eating Desert Time EDT(n)	Exit Time min
Std 1	0.454		0.516	0.922	0.405	0.965	0.686	0.623	0.327
	28	0.47	MC	0	39	29	ND	0	30.1
Std 2	0.046		0.239	0.356	0.686	0.577	0.234	0.439	0.588
	39	0.65	SB	51	0	21	ND	0	22.5
Std 3	0.024		0.034	0.134	0.534	0.648	0.244	0.525	0.34
	49	0.82	SB	38	0	22	ND	0	23.5
Std 4	0.162		0.032	0.224	0.209	0.441	0.493	0.85	0.607
	65	1.08	SB	43	0	18	ND	0	19.8
Std 5	0.359		0.946	0.607	0.42	0.058	0.197	0.336	0.353
	89	1.48	MC	0	40	11	ND	0	13.2
Std 6	0.908		0.385	0.181	0.683	0.067	0.856	0.736	0.328
	135	2.25	SB	40	0	11	ND	0	13.9
Std 7	0.287		0.537	0.196	0.087	0.297	0.772	0.564	0.633
	156	2.60	MC	0	20	15	ND	0	17.9
Std 8	0.98		0.383	0.485	0.909	0.061	0.201	0.356	0.361
	205	3.42	SB	59	0	11	ND	0	15.4
Std 9	0.253		0.671	0.545	0.765	0.651	0.03	0.839	0.546
	225	3.75	MC	0	60	23	YD	11	27.9
Std 10	0.16		0.498	0.09	0.432	0.187	0.588	0.248	0.954
	241	4.02	MC	0	40	13	ND	0	17.7

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Number of students are still in the cafeteria = 10

Percentage students at the sandwich bar= 5/10 = 0.5

Percentage students at the main course counter = 5/ 10 = 0.5

Percentage of students on tables = 10/10 = 1

Percentage of students take dessert = 1/10

Average time a student with sandwich spends in the cafeteria

= (total time for prep + Total eating time) / 5

=[(51+38+43+40+59)+60*(21+22+18+11+11)]/5 = 5211/5 = 1042.2 Sec= 17.4 min

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Question #5: **(20 point)**

A traffic department of a small city wants to simulate the occurrence time for the accident in an intersection using some different choices of distributions to estimate the number of accidents per month (30 days). Use the following U(0,1) numbers to compute the simulation under the following assumptions.

1. Estimate the number of accidents in the 1st month assuming the time between accident is a _truncated_ exponential distribution with between (2 and 10) and mean 4 days
2. Estimate the number of accidents in the 1st month assuming the time between accident is Erlang distribution with parameters $k = 3$ and $\lambda = 0.75$.
3. Estimate the number of accidents in the 1st month assuming the time between accident is negative binomial (R.V. X be number of trials until success) with parameters $k = 3$ and $p(success) = 0.65$.

		truncExp (2 and 10)	Day
1	0.737	6.06	6.06
2	0.293	3.17	9.22
3	0.136	2.50	11.73
4	0.848	7.29	19.01
5	**0.692**	**5.65**	**24.66**
6	0.727	5.96	30.62
7	0.116	2.42	33.04

$$W = ((F(10) - F(2)U) + F(2))$$

$$F(10) = 1 - e^{-\frac{10}{4}} = 0.917$$

$$F(2) = 1 - e^{-\frac{2}{4}} = 0.393$$

$$W = ((0.917 - 0.393)U + 0.393)$$

$$W = 0.524 U + 0.393$$

$$F^{-1}(w) = -4\ln(1-w)$$

		Exp(0.75)	Er(3,0.75)	Day	Acc#
1	0.737	1.78			
2	0.293	0.46			
3	0.136	0.19	**2.44**	**2.44**	**1**
4	0.848	2.51			
5	0.692	1.57			
6	0.727	1.73	**5.81**	**8.25**	**2**
7	0.116	0.16			
8	0.074	0.10			
9	0.262	0.41	**0.67**	**8.92**	**3**
10	0.385	0.65			
11	0.317	0.51			
12	0.923	3.42	**4.58**	**13.50**	**4**
13	0.057	0.08			
14	0.441	0.78			
15	0.454	0.81	**1.66**	**15.16**	**5**
16	0.346	0.57			
17	0.024	0.03			
18	0.162	0.24	**0.83**	**15.99**	**6**
19	0.359	0.59			
20	0.908	3.18			
21	0.287	0.45	**4.23**	**20.22**	**7**
22	0.98	5.22			
23	0.253	0.39			
24	**0.16**	**0.23**	**5.84**	**26.06**	**8**

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25	0.815	2.25			
26	0.5	0.92			
27	0.872	2.74	**5.92**	**31.97**	**NO**

		Geo 0.65	NegBin (3,0.65)	Day	Acc#
1	0.737	1			
2	0.293	0			
3	0.136	0	**1**	**1**	**1**
4	0.848	1			
5	0.692	1			
6	0.727	1	**3**	**4**	**2**
7	0.116	0			
8	0.074	0			
9	0.262	0	**0**	**4**	**3**
10	0.385	0			
11	0.317	0			
12	0.923	2	**2**	**6**	**4**
13	0.057	0			
14	0.441	0			
15	0.454	0	**0**	**6**	**5**
16	0.346	0			
17	0.024	0			
18	0.162	0	**0**	**6**	**6**
19	0.359	0			
20	0.908	2			
21	0.287	0	2	**8**	7
22	0.98	3			
23	0.253	0			
24	0.16	0	**3**	**11**	**8**
25	0.815	1			
26	0.5	0			
27	**0.872**	1	**2**	**13**	**9**
28	0.99	4			
29	0.516	0			
30	0.239	0	**4**	**17**	

		Ber(0.65)	Geo(0.75)	Neg	Day	Acc#
1	0.737	0				
2	0.293	1	2			
3	0.136	1	1			
4	0.848	0				
5	0.692	0				
6	0.727	0				
7	0.116	1	4	7	7	1
8	0.074	1	1			
9	0.262	1	1			
10	0.385	1	1	3	10	2
11	0.317	1	1			
12	0.923	0				
13	0.057	1	2			
14	0.441	1	1	4	14	3
15	0.454	1	1			
16	0.346	1	1			
17	0.024	1	1	3	17	4
18	0.162	1	1			
19	0.359	1	1			
20	0.908	0				
21	0.287	1	2	4	21	5
22	0.98	0				
23	0.253	1	2			
24	0.16	1	1			
25	0.815	0				
26	**0.5**	**1**	**2**	**5**	**26**	**6**
27	0.872	0				
28	0.993	0				
29	0.516	1	3			
30	0.239	1	1			
31	0.034	1	1	5	31	No

...

14 accidents