King Saud University
College of Sciences
Mathematics Department

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Solution of Quiz 2 December 2, 2019 ACTU 464

Question (**5 marks**)

A portfolio of independent insurance policies has three classes of policies:

Class	Number in Class	Probability of Claim per Policy	Claim Amount b_k
1	1000	0.1	1
2	2000	0.2	2
3	500	0.3	3

1. Calculate the expectation and variance of the aggregate loss S.
2. Use normal approximation to calculate θ such that the probability of that the aggregate loss is less or equal than the $\Pi_{\mathrm{SL}}(\theta)$ is equal to 0.95.
3. Find $\Pi_{\mathrm{SL}}(\theta)$.

Solution

1. We have $\mathrm{E}[S] = \sum_{i=1}^{3} n_k\ b_k\ q_k = 1000 \times 1 \times 0.1 + 2000 \times 2 \times 0.2 + 500 \times 3 \times 0.3 = \mathbf{1350}$. And

$$\begin{aligned} \sigma_S^2 &= \mathrm{Var}(S) = \sum_{i=1}^{3} n_k\ b_k^2\ q_k\,(1-q_k) \\ &= 1000 \times 1^2 \times 0.1 \times 0.9 + 2000 \times 2^2 \times 0.2 \times 0.8 + 500 \times 3^2 \times 0.3 \times 0.7 = \mathbf{2315}. \end{aligned}$$

2. Under normal approximation the r.v. $T = \frac{S-\mathrm{E}[S]}{\sigma_S}$ follows a standard normal distribution, therefore

$$\mathrm{P}\left(S \le \Pi_{\mathrm{SL}}(\theta)\right) = \mathrm{P}\left(\frac{S-\mathrm{E}[S]}{\sigma_S} \le \frac{\Pi_{\mathrm{SL}}(\theta)-1350}{\sqrt{2315}} = \theta\frac{1350}{\sqrt{2315}}\right) = 0.95,$$

hence $\theta = \frac{1.644854 \times \sqrt{2315}}{1350} = \mathbf{0.058623}$.

3. The safety loading premium is $\Pi_{\mathrm{SL}}(0.058623) = 1.058623 \times 1350 = \mathbf{1429.141}$.