الضوابط التنظيمية للاختبارات المنزلية Take Home Exam الخاصة بالطلاب.

1- على الطلاب ضرورة الاطلاع على كل ما يخص الاختبارات من تعليمات سواء عن طريق البريد الإلكتروني وصفحة التعليمات في الاختبار.

2- الالتزام بوقت بداية الاختبار المدرجة في البوابة الاكاديمية ونهاية الاختبار التي ستحدد من قبل عضو هيئة التدريس.

3- تسليم اجابة الاختبار دون تأخير. ومن يتأخر عن موعد التسليم لن يقبل منه.

4- تسليم الإجابة عبر البلاك بورد أو البريد الإلكتروني أو كليهما (إما مستند pdf أو صورة ممسوحة ضوئياً).

5- يجب أن يتم إتمام الاختبار بشكل فردي. ويُحظر عرضه أو مناقشته مع أي شخص آخر، بما في ذلك (على سبيل المثال لا الحصر) الطلاب الآخرين في نفس المقرر.

6- يمكن للطالب استخدام أي مادة متاحة يريدها، بما في ذلك العروض التقديمية ومذكرة المحاضرات والكتب والإنترنت، ولا يجب نسخ المعلومة كما هي ولكن تكتب حسب فهم الطالب وإلا ستعتبر إقتباسا يؤثر على درجة الطالب.

9- سوف يتم النظر في جميع الحالات الطلابية التي لم يتمكنوا من أداء الاختبار المنزلي بعقد اختبار بديل لها في بداية الفصل الدراسي الأول من العام ١٤٤٢هـ.

Exercise 1 *For a portfolio of independent risks, you are given:*
(i) The risks are divided into two classes, class A and class B.
(ii) Equal numbers of risks are in class A and class B.
(iii) For each class, the probability of having exactly one claim during the year is 10% *and the probability of having* 0 *claim is* 90%*. All claims for class A are of size* 4*. All claims for class B are of size d, an unknown quantity.*
An individual from a randomly chosen risk class is observed for one year and the total loss is observed. We want to determine the Buhlmann credibility estimate for the expected loss in the following year. Calculate the limit of the Buhlmann credibility factor (Z) as d approaches infinity.

Exercise 2 *For a portfolio of independent risks, you are given:*
(i) The risks are divided into two classes, class A and class B.
(ii) Equal numbers of risks are in class A and class B.
(iii) For each risk in class A, each period's losses are exponentially distributed with mean 1*. For each risk in class B, each period's losses are exponentially distributed with mean* β*. A risk is selected at random for one period and is found that losses equal* 1*. Calculate the limit of the Buhlmann credibility estimate for losses of the second period as the value of* β *approaches* 0*.*

Exercise 3 *Given a first observation with a value of* 3*, the Buhlmann credibility estimate for the expected value of the second observation is* $5/2$*. Given the first two observations each having value of* 3*, the Buhlmann credibility estimate for the expected value of the third observation is* $8/3$*. Find the Buhlmann credibility estimate for the expected value of the fourth observation if the first three observations are each* 3*.*

Exercise 4 *You are given:*
(i) The annual number of claims X *for each policyholder is a random variable with pmf* $Pr(X = x \,|\Theta = \theta\,) = \theta(1-\theta)^x$, $x = 0, 1, 2, ...$
(ii)The distribution of θ *across all policyholders has probability density function:*

$$u(\theta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\theta^{\alpha-1}(1-\theta)^{\beta-1}, 0 < \theta < 1.$$

(a) Calculate the hypothetical mean $E(X\,|\Theta = \theta\,)$*.*
(b) Calculate the collective premium $E(X)$*.*
(c) Calculate the posterior distribution of Θ *after n years observations* $X_1, ..., X_n$*.*

Credibility ACTU 465
final exam.

Ex 1

$$\begin{cases} 0.9 & X=0 \\ 0.1 & \begin{cases} A & X=4 \\ B & X=d \end{cases} \end{cases}$$

$$k = \frac{v}{a} = \frac{E(V(X|\theta))}{V(E(X|\theta))}$$

$$\mu(\theta) = E(X|\theta) \begin{cases} 0.1 \times 4 & \text{for } \theta = A \\ 0.1 \times d & \text{for } \theta = B \end{cases}$$

$$\mu = E(\mu(\theta)) = \frac{0.1}{2}(4+d).$$

$$\mu^2(\theta) \begin{cases} 0.1^2 \times 4^2 & \text{for } \theta = A \\ 0.1^2 \times d^2 & \text{for } \theta = B \end{cases}$$

$$E(\mu^2(\theta)) = \frac{0.1^2}{2}(4^2+d^2).$$

$$a = E(\mu^2(\theta)) - \mu^2 = \frac{0.1^2}{2}(4^2+d^2) - \frac{0.1^2}{4}(4+d)^2$$

$$= \frac{0.1^2}{4}\big(\underbrace{32+2d^2-16-d^2-8d}_{(d-4)^2}\big)$$

$$= 0.0025d^2 - 0.02d + 0.04$$

$$V(X|\theta) = E(X^2|\theta) - \mu^2(\theta)$$

$$= \begin{cases} 0.1 \times 16 - 0.1^2 \times 16 = 0.09 \times 16 = 1.44 & (\theta = A) \\ 0.1 \times d^2 - 0.1^2 \times d^2 = 0.09 \times d^2 & (\theta = B) \end{cases}$$

$$v = \frac{0.09}{2}(16+d^2) = 0.72 + 0.045d^2$$

$$k = \frac{0.09}{2}(16+d^2) \times \frac{4}{(0.1)^2(d-4)^2} = 2\,\frac{0.09}{0.01}\,\frac{16+d^2}{(d-4)^2}$$

$$k \xrightarrow[d\to+\infty]{} 18 \,(??) \Rightarrow Z = \frac{1}{1+k} \xrightarrow[k\to+\infty]{} \frac{1}{19}$$

$$\lim_{d\to+\infty} k = \frac{2(0.09)}{0.01}\left(\frac{1}{1}\right) = 18$$

Ex 2

$$\frac{1}{2}: X \sim \text{Exp}(\text{mean}=1) \quad \text{for } \theta = A$$

$$\frac{1}{2}: X \sim \text{Exp}(\text{mean}=\beta) \quad \text{for } \theta = B$$

$$BC = Z \times 1 + (1-Z) \times \mu$$

$$\mu(\theta) = E(X|\theta) = \begin{cases} 1 & (\theta = A) \\ \beta & (\theta = B) \end{cases}$$

$$\mu = E(\mu(\theta)) = \frac{1}{2}(1+\beta) \xrightarrow[\beta \to 0]{} 1/2$$

$$v(\theta) = V(X|\theta) = \begin{cases} 1 & (\theta = A) \\ \beta^2 & (\theta = B) \end{cases}$$

$$v = E(v(\theta)) = \frac{1}{2}(1+\beta^2)$$

$$a = V(\mu(\theta)) = \frac{1}{2}(1+\beta^2) - \frac{1}{4}(1+\beta)^2$$

$$= \frac{1}{4}\left(2 + 2\beta^2 - 1 - \beta^2 - 2\beta\right) = \frac{1}{4}(\beta-1)^2$$

$$k = \frac{v}{a} = \frac{2(1+\beta^2)}{(\beta-1)^2} \xrightarrow[\beta \to 0]{} 2$$

$$\Rightarrow Z = \frac{1}{1+k} \xrightarrow[\beta \to 0]{} 1/3$$

$$BC = Z \times 1 + (1-Z) \times \mu \xrightarrow[\beta \to 0]{} \frac{1}{3}(1) + \frac{2}{3}\left(\frac{1}{2}\right) = \frac{2}{3}$$

Ex 3

$$BC(t=2) = Z_1 \times 3 + (1-Z_1)\times \mu = 5/2$$

with $Z_1 = \frac{1}{1+k}$

$$BC(t=3) = Z_2 \times 3 + (1-Z_2)\times \mu = 8/3$$

with $Z_2 = \frac{2}{2+k}$

$$\Rightarrow \begin{cases} \frac{1}{1+k} 3 + \frac{k}{1+k} \mu = 5/2 \\ \frac{2}{2+k} 3 + \frac{k}{2+k} \mu = 8/3 \end{cases}$$

$$\Rightarrow \begin{cases} 3 + k\mu = \frac{5}{2}(1+k) & (1) \\ 6 + k\mu = \frac{8}{3}(2+k) & (2) \end{cases}$$

$$(2)-(1) \Rightarrow 3 = \frac{16}{3} + \frac{8}{3}k - \frac{5}{2} - \frac{5}{2}k$$

$$\Rightarrow 3 = \frac{17}{6} + \frac{k}{6} \Rightarrow \frac{k}{6} = \frac{1}{6} \Rightarrow k = 1$$

$$(1) \Rightarrow \mu = \frac{5}{2}(2) - 3 = 2$$

$$BC(t=4) = Z_3 \times 3 + (1-Z_3)\times \mu$$

with $Z_3 = \frac{3}{3+k} = \frac{3}{4}$

$$\Rightarrow BC(t=4) = \frac{3}{4}(3) + \frac{1}{4}(2) = \frac{11}{4} = 2.75$$

Ex 4 (Bonus)

(a) $$E(X|\theta) = \sum_0^\infty x\,P(x|\theta) = \sum_0^\infty x\theta(1-\theta)^x$$

$$= \theta(1-\theta)\sum_0^\infty x(1-\theta)^{x-1} = -\theta(1-\theta)\left(\sum_0^\infty (1-\theta)^x\right)'$$

$$= -\theta(1-\theta)\left(\frac{1}{1-(1-\theta)}\right)' = -\theta(1-\theta)\left(-1/\theta^2\right)$$

$$= \frac{1-\theta}{\theta}.$$

(b) $$E(X) = E(E(X|\theta)) = \int \frac{1-\theta}{\theta}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\theta^{\alpha-1}(1-\theta)^{\beta-1}d\theta$$

$$= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\int \theta^{\alpha-2}(1-\theta)^{\beta}d\theta = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\frac{\Gamma(\alpha-1)\Gamma(\beta+1)}{\Gamma(\alpha+\beta)}$$

$$= \frac{\Gamma(\alpha-1)}{\Gamma(\alpha)}\frac{\Gamma(\beta+1)}{\Gamma(\beta)}.$$

Using $\boxed{\Gamma(x+1) = x\,\Gamma(x)}$ (not $\Gamma(\alpha) = (\alpha-1)!$ since α may not belong to natural number set $\{1, 2, \dots\}$)

we have $E(X) = \dfrac{\beta}{\alpha-1}$

(c) $$u(\theta|x_1,\dots,x_n) = \frac{P(x_1,\dots,x_n|\theta)u(\theta)}{P(x_1,\dots,x_n)} = \frac{\theta^n(1-\theta)^{\sum_1^n x_i}u(\theta)}{P(x_1,\dots,x_n)}$$

$$\simeq \theta^{\alpha+n-1}(1-\theta)^{\beta+\sum x_i-1}$$

Hence
$$\boxed{u(\theta|x_1,\dots,x_n) = \frac{\Gamma\left(\alpha+n+\beta+\sum_1^n x_i\right)}{\Gamma(\alpha+n)\Gamma\left(\beta+\sum_1^n x_i\right)}\theta^{\alpha+n-1}(1-\theta)^{\beta+\sum x_i-1}}$$

$\Theta|_{x_1,\dots,x_n}$ is Beta $\left(\hat{\alpha} = \alpha+n,\ \hat{\beta} = \beta+\sum_1^n x_i\right)$