كلية العلوم
قسم الإحصاء وبحوث العمليات

College of Science.
Department of Statistics & Operations Research

Final Exam
Academic Year 1442-1443 Hijri- SecondSemester

Exam Information معلومات الامتحان			
Course name	نظرية المصداقية		اسم المقرر
Course Code	465 ريك		رمز المقرر
Exam Date	2021-04-25	1443-09-13	تاريخ الامتحان
Exam Time	09: 00 AM		وقت الامتحان
Exam Duration	3 hours	ثلاث ساعات	مدة الامتحان
Classroom No.			رقم قاعة الاختبار
Instructor Name			اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of 1 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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تعليمات عامة:

- عدد صفحات الامتحان 1 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف و الساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة
This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1				
2				
3				
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EXAM COVER PAGE

Exercise 1 *For a particular policy, the conditional probability of the annual number of claims given $\Theta = \theta$ ($\theta = 0, 1$) are as follows:*
For $\theta = 0$,

Number of claims	0	1	2
Probability	4/6	1/6	1/6

and for $\theta = 1$,

Number of claims	0	1	2
Probability	1/4	1/2	1/4

The probability distribution of θ *is*

θ	0	1
Probability	0.6	0.4

For a sample of 10 *years, a total of* 10 *claims has been observed. Calculate the Buhlmann credibility estimate of the number of claims in Year* 11.

0.82 0.86 0.89 0.94 0.98

Exercise 2 *You are given:*
(i) The probability that an insured will have at least one loss during any year is p.
(ii) The prior distribution for p *is uniform on* $[0, 0.75]$.
(iii) An insured is observed for 14 *years and has at least one loss every year.*
Calculate the posterior probability that the insured will have at least one loss during Year 15.

0.403 0.503 0.603 0.703 0.803

Hint: consider $Y_i = 1$ *if at least one loss occurs during year* i, *and* 0 *otherwise.*

Exercise 3 *Data for a policyholder is known for the past* 30 *years. During that time, the policyholder's average losses per year were* 100. *To determine full credibility, you select the values* $k = 0.05$ *and* $p = 0.95$. *The standard deviation of losses in each year is* 30. *Use limited fluctuation credibility theory to determine whether these data are fully credible.*

Ex1

X	0	1
0	4/6	1/4
1	1/6	2/4
2	1/6	1/4

$\pi(\theta) = \begin{cases} 0.6 & \theta = 0 \\ 0.4 & \theta = 1 \end{cases}$

$\boxed{\text{Final sol}}$

$\mu(\theta) = \begin{cases} 1/2 \\ 1 \end{cases}$ $\quad \mu = E(\mu(\theta)) = 0.3 + 0.4 = 0.7$

$a = V(\mu(\theta)) = E(\mu^2(\theta)) - \mu^2$

$= \frac{1}{4}(0.6) + 1(0.4) - 0.7^2 = 0.06$

$v = E(\mu(\theta))$

$v(\theta) = V(x|\theta) = \begin{cases} E(x^2/\theta) \\ E(x^2(\theta)) - \mu(\theta)^2 = \end{cases} \begin{cases} 5/6 - 1/4 = 7/12 \\ 6/4 - 1 = 1/2 \end{cases}$

$v = 0.55$

0.86

$k = \frac{0.55}{0.06} = 9.166 = 55/6$

$Z = \frac{10}{10+k} = 0.52 = \frac{60}{115}$

$B.C = Z.\bar{x} + (1-Z)\mu$

$= Z(1) + (1-Z)\,0.7$

$= 0.856 \simeq 0.86$

Ex2 $Y = \begin{cases} 1 & X \geq 1 \\ 0 & X = 0 \end{cases}$ $f(p) = \frac{1}{3/4} = 4/3$; $0 \leq p \leq 3/4$

$\pi[X \geq 1 \mid x_1 \geq 1 \ldots x_{14} \geq 1] = E[Y=1 \mid y_1 \ldots y_{14} = 1]$

$= \int_0^{3/4} E(Y=1|p)\, \pi(p \mid y_1 \ldots y_{14}=1)\, dp$

$\pi(p \mid y_1 \ldots y_{14}=1) = \frac{f(y_1 = \ldots y_{14} = 1|p)\, \pi(p)}{\int \quad}$

$= \frac{p^{14} \times \cancel{4/3}}{\int_0^{3/4} \cancel{4/3}\, p^{14}\, dp} = \frac{p^{14}}{\frac{(3/4)^{15}}{15}} = \frac{15\, p^{14}}{(3/4)^{15}}$

$E[\quad] = \int_0^{3/4} \frac{15\, p^{14}}{(3/4)^{15}} (p)\, dp = \frac{15}{(3/4)^{15}} \left[\frac{p^{16}}{16}\right]_0^{3/4}$

$= \frac{15}{16}(3/4) = 0.703$

Ex3 $n = 30$; $k = 0.05$; $z = 1.96$ $\sigma_x = 30$; $\mu = 100$

$n_f = \lambda_0 C_x^2 = \left(\frac{1.96}{0.05}\right)^2 \times \frac{30^2}{100^2} = 136$

$n < n_f \Rightarrow$ no full credibility