King Saud University

College of Science.
Department of Statistics & Operations Research

كلية العلوم
قسم الإحصاء وبحوث العمليات

First Midterm Exam
Academic Year 1442-1443 Hijri- First Semester

Exam Information معلومات الامتحان			
Course name	نظرية المصداقية		اسم المقرر
Course Code	465 ريك		رمز المقرر
Exam Date	2020-10-26	1442-03-10	تاريخ الامتحان
Exam Time	10: 00 AM		وقت الامتحان
Exam Duration	2 hours	ساعتان	مدة الامتحان
Classroom No.			رقم قاعة الاختبار
Instructor Name			اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of 4 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
-

تعليمات عامة:

- عدد صفحات الامتحان 4 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف و الساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1				
2				
3				
4				
5				
6				
7				
8				

EXAM COVER PAGE

Exercise 1 *The criterion for the number of exposures needed for full credibility is changed from requiring* $\bar{X}$ *to be within* $0.05E(\bar{X})$ *with probability* 0.90*, to requiring* $\bar{X}$ *to be within* $kE(\bar{X})$ *with probability* 0.95*. Find the value of* k *that results in no change in the standard for full credibility for number of exposures of* X*.*

a) 0.0524 *b)* 0.0548 *c)* 0.0572 *d)* 0.0596 *e)* 0.0620

Exercise 2 *Total claim amount per period* S *follows a compound Poisson claims distribution. The standard for full credibility for total claims in a period* S *based on number of claims is* 1500 *claims. It is then discovered that an incorrect value of the coefficient of variation for the severity distribution* Y *was used to determine the full credibility standard. The original coefficient of variation used was* 0.6211*, but the corrected coefficient of variation for* Y *is* 0.5200*. Find the corrected standard for full credibility for* S *based on number of claims.*

a) 1300 *b)* 1325 *c)* 1350 *d)* 1375 *e)* 1400

Exercise 3 *Let* S_j *the total losses experienced by a policyholder at period* $j = 1, ..., n$ *and* S_j *is a coumpound Poisson and loss amounts have mean* 5 *and variance* 100*. Determine the expected total number of claims required for full credibilty if*
a) The aggregate losses must be within 3% *of expected aggregate losses* 95% *of the time.*
b) The actual number of claims must be within 3% *of the expected number of claims with probability of* 95%*.*

Exercise 4 *We have:*
(i) $S = \sum_{j=1}^{N} X_j$ *and the* X_j *are independent and independent of* N*.*
(ii) X_j *is a Pareto distribution with parameters* $(3, 3)$*.*
(iii) N *is negative binomial with parameters* $(r, 2)$*.*
Calculate the minimum value that r *must have for* S *to be within* 5% *of the expected value with* 90%*. You may use the normal approximation.*

Exercise 5 *Assume there are two different types of drivers, good (G) and bad drivers (B). The variable* X *is the number of claims in any one year.*

x	$P(x \mid G)$	$P(x \mid B)$	
0	0.7	0.5	$P(G) = 0.75$
1	0.2	0.3	$P(B) = 0.25$
2	0.1	0.2	

Suppose a policyholder had 0 *claims the first year and* 1 *claim the second year. Determine*
a) The posterior probability $\pi(G \mid x_1, x_2)$
b) Determine the Bayesian estimate of this insured's claim count in the next (third) policy year.

Exercise 6 *A risk class is made up of three equally sized groups of individuals. Groups are classified as Type A, Type B and Type C. Any individual of any type has probability of* 0.5 *of having no claim in the coming year and has a probability of* 0.5 *of having exactly 1.*

$$P(\textit{claim of amount } x \,|\, \textit{Type A and a claim occurs}) = \begin{cases} 2/3 & x = 1 \\ 1/3 & x = 2 \end{cases}$$

$$P(\textit{claim of amount } x \,|\, \textit{Type B and a claim occurs}) = \begin{cases} 1/2 & x = 1 \\ 1/2 & x = 2 \end{cases}$$

$$P(\textit{claim of amount } x \,|\, \textit{Type C and a claim occurs}) = \begin{cases} 5/6 & x = 1 \\ 1/6 & x = 2 \end{cases}$$

An insured is chosen at random from the risk class and is found to have a claim of amount 2 in Year 1. Determine the Bayesian estimate of this insured's claim amount in the next policy year.

a) $\frac{25}{36}$ *b)* $\frac{3}{4}$ *c)* $\frac{29}{36}$ *d)* $\frac{11}{12}$

A.2.3 Two-parameter distributions

A.2.3.1 Pareto (Pareto Type II, Lomax)—α, θ

$$
\begin{aligned}
f(x) &= \frac{\alpha\theta^\alpha}{(x+\theta)^{\alpha+1}} \qquad F(x) = 1 - \left(\frac{\theta}{x+\theta}\right)^\alpha \\
\mathrm{E}[X^k] &= \frac{\theta^k\Gamma(k+1)\Gamma(\alpha-k)}{\Gamma(\alpha)}, \quad -1 < k < \alpha \\
\mathrm{E}[X^k] &= \frac{\theta^k k!}{(\alpha-1)\cdots(\alpha-k)}, \quad \text{if } k \text{ is an integer} \\
\mathrm{VaR}_p(X) &= \theta[(1-p)^{-1/\alpha} - 1] \\
\mathrm{TVaR}_p(X) &= \mathrm{VaR}_p(X) + \frac{\theta(1-p)^{-1/\alpha}}{\alpha-1}, \quad \alpha > 1 \\
\mathrm{E}[X \wedge x] &= \frac{\theta}{\alpha-1}\left[1 - \left(\frac{\theta}{x+\theta}\right)^{\alpha-1}\right], \quad \alpha \neq 1 \\
\mathrm{E}[X \wedge x] &= -\theta \ln\left(\frac{\theta}{x+\theta}\right), \quad \alpha = 1 \\
\mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k\Gamma(k+1)\Gamma(\alpha-k)}{\Gamma(\alpha)}\beta[k+1, \alpha-k; x/(x+\theta)] + x^k\left(\frac{\theta}{x+\theta}\right)^\alpha, \quad \text{all } k \\
\text{mode} &= 0
\end{aligned}
$$

A.2.3.2 Inverse Pareto—τ, θ

$$
\begin{aligned}
f(x) &= \frac{\tau\theta x^{\tau-1}}{(x+\theta)^{\tau+1}} \qquad F(x) = \left(\frac{x}{x+\theta}\right)^\tau \\
\mathrm{E}[X^k] &= \frac{\theta^k\Gamma(\tau+k)\Gamma(1-k)}{\Gamma(\tau)}, \quad -\tau < k < 1 \\
\mathrm{E}[X^k] &= \frac{\theta^k(-k)!}{(\tau-1)\cdots(\tau+k)}, \quad \text{if } k \text{ is a negative integer} \\
\mathrm{VaR}_p(X) &= \theta[p^{-1/\tau} - 1]^{-1} \\
\mathrm{E}[(X \wedge x)^k] &= \theta^k\tau\int_0^{x/(x+\theta)} y^{\tau+k-1}(1-y)^{-k}dy + x^k\left[1 - \left(\frac{x}{x+\theta}\right)^\tau\right], \quad k > -\tau \\
\text{mode} &= \theta\frac{\tau-1}{2}, \quad \tau > 1, \text{ else } 0
\end{aligned}
$$

A.2.3.3 Loglogistic (Fisk)—γ, θ

$$
\begin{aligned}
f(x) &= \frac{\gamma(x/\theta)^\gamma}{x[1+(x/\theta)^\gamma]^2} \qquad F(x) = u, \quad u = \frac{(x/\theta)^\gamma}{1+(x/\theta)^\gamma} \\
\mathrm{E}[X^k] &= \theta^k\Gamma(1+k/\gamma)\Gamma(1-k/\gamma), \quad -\gamma < k < \gamma \\
\mathrm{VaR}_p(X) &= \theta(p^{-1} - 1)^{-1/\gamma} \\
\mathrm{E}[(X \wedge x)^k] &= \theta^k\Gamma(1+k/\gamma)\Gamma(1-k/\gamma)\beta(1+k/\gamma, 1-k/\gamma; u) + x^k(1-u), \quad k > -\gamma \\
\text{mode} &= \theta\left(\frac{\gamma-1}{\gamma+1}\right)^{1/\gamma}, \quad \gamma > 1, \text{ else } 0
\end{aligned}
$$

B.2.1.2 Geometric—β

$$
\begin{aligned}
p_0 &= \frac{1}{1+\beta}, \quad a = \frac{\beta}{1+\beta}, \quad b = 0 \qquad p_k = \frac{\beta^k}{(1+\beta)^{k+1}} \\
\mathrm{E}[N] &= \beta, \quad \mathrm{Var}[N] = \beta(1+\beta) \qquad P(z) = [1-\beta(z-1)]^{-1}.
\end{aligned}
$$

This is a special case of the negative binomial with $r = 1$.

B.2.1.3 Binomial—$q, m, (0 < q < 1,\ m$ an integer$)$

$$
\begin{aligned}
p_0 &= (1-q)^m, \quad a = -\frac{q}{1-q}, \quad b = \frac{(m+1)q}{1-q} \\
p_k &= \binom{m}{k} q^k (1-q)^{m-k}, \quad k = 0, 1, \ldots, m \\
\mathrm{E}[N] &= mq, \quad \mathrm{Var}[N] = mq(1-q) \qquad P(z) = [1+q(z-1)]^m.
\end{aligned}
$$

B.2.1.4 Negative binomial—β, r

$$
\begin{aligned}
p_0 &= (1+\beta)^{-r}, \quad a = \frac{\beta}{1+\beta}, \quad b = \frac{(r-1)\beta}{1+\beta} \\
p_k &= \frac{r(r+1)\cdots(r+k-1)\beta^k}{k!(1+\beta)^{r+k}} \\
\mathrm{E}[N] &= r\beta, \quad \mathrm{Var}[N] = r\beta(1+\beta) \qquad P(z) = [1-\beta(z-1)]^{-r}.
\end{aligned}
$$

B.3 The $(a, b, 1)$ class

To distinguish this class from the $(a, b, 0)$ class, the probabilities are denoted $\Pr(N = k) = p_k^M$ or $\Pr(N = k) = p_k^T$ depending on which subclass is being represented. For this class, p_0^M is arbitrary (that is, it is a parameter) and then p_1^M or p_1^T is a specified function of the parameters a and b. Subsequent probabilities are obtained recursively as in the $(a, b, 0)$ class: $p_k^M = (a + b/k)p_{k-1}^M$, $k = 2, 3, \ldots$, with the same recursion for p_k^T There are two sub-classes of this class. When discussing their members, we often refer to the "corresponding" member of the $(a, b, 0)$ class. This refers to the member of that class with the same values for a and b. The notation p_k will continue to be used for probabilities for the corresponding $(a, b, 0)$ distribution.

B.3.1 The zero-truncated subclass

The members of this class have $p_0^T = 0$ and therefore it need not be estimated. These distributions should only be used when a value of zero is impossible. The first factorial moment is $\mu_{(1)} = (a+b)/[(1-a)(1-p_0)]$, where p_0 is the value for the corresponding member of the $(a, b, 0)$ class. For the logarithmic distribution (which has no corresponding member), $\mu_{(1)} = \beta/\ln(1+\beta)$. Higher factorial moments are obtained recursively with the same formula as with the $(a, b, 0)$ class. The variance is $(a+b)[1-(a+b+1)p_0]/[(1-a)(1-p_0)]^2$.For those members of the subclass which have corresponding $(a, b, 0)$ distributions, $p_k^T = p_k/(1-p_0)$.

Mid 1
Credibility Sol.

Ex 1 $\quad n_y = \lambda_0^1 C_x^2 = \lambda_0^2 C_x^2$

$\Leftrightarrow \lambda_0^1 = \lambda_0^2 \Leftrightarrow \left(\frac{1.645}{0.05}\right)^2 = \left(\frac{1.96}{k}\right)^2$

$\Leftrightarrow k = 0.05\left(\frac{1.96}{1.645}\right) = 0.0569.$

Ex 2 $\quad \left(\sum N_i\right)_1 = \lambda_0\left(1 + C_{Y_1}^2\right)$

$\Leftrightarrow 1500 = \lambda_0\left(1 + (0.6211)^2\right)$

$\Leftrightarrow \lambda_0 = \frac{1500}{1 + (0.6211)^2}.$

$\left(\sum N_i\right)_2 = \lambda_0\left(1 + C_{Y_2}^2\right)$

$= \frac{1500}{1 + (0.6211)^2}\left(1 + 0.52^2\right) = 1375$

Ex 3 $\quad S_1 \; S_2 \; \ldots \; S_N$ ~~with~~S

with $S = \sum_1^N x_i \qquad \mu_x = 5 \; ; \; \sigma_x^2 = 100$

(a) $\left(\sum N_i\right)^f = \lambda_0\left(1 + C_Y^2\right)$

$= \left(\frac{1.96}{0.03}\right)^2\left(1 + \frac{100}{25}\right) = 21{,}342$

(b) $\left(\sum N_i\right)^f = \lambda_0 = \left(\frac{1.96}{0.03}\right)^2 = 4{,}268.$

Ex 4 $S = \sum_{1}^{N} X_j$ with $X \sim \text{Pareto}(3,3)$

$\mu_X = 3/2$, $VX = 27/4$.

$N \sim \text{Neg. Bin}(r, 2)$ $\mu_N = 2r$; $\sigma_N^2 = 6r$.

$P\left\{ -k < \frac{S - \mu_S}{\mu_S} < k \right\} \geq p$

$\Leftrightarrow P\left\{ -k \frac{\mu_S}{\sigma_S} < \frac{S - \mu_S}{\sigma_S} < k \frac{\mu_S}{\sigma_S} \right\} \geq p$.

$\Leftrightarrow \frac{k \mu_S}{\sigma_S} = z_{\alpha/2}$

$\mu_S = \mu_N \mu_X = 3r$

$\sigma_S^2 = \sigma_N^2 \mu_X^2 + \mu_N \sigma_X^2$

$= (6r)(9/4) + (2r)(27/4)$

$= 27r$

$\frac{k \mu_S}{\sigma_S} = \frac{k(3r)}{\sqrt{27r}} = z$

$\Leftrightarrow \frac{9k^2 r^2}{27r} = z^2 \Leftrightarrow r = 3\left(\frac{z}{k}\right)^2 = 3\lambda_0$

$= \left(\frac{1.645}{0.05}\right)^2 = 3,247.23$

Ex 5

(a) $\pi(G \mid x_1, x_2) = \frac{\ell(x_1 = 0, x_2 = 1 \mid G)\, \pi(G)}{\ell(x_1 = 0, x_2 = 1)}$

$= \frac{(0.7)(0.2)(0.75)}{(0.7)(0.2)(0.75) + (0.5)(0.3)(0.25)}$

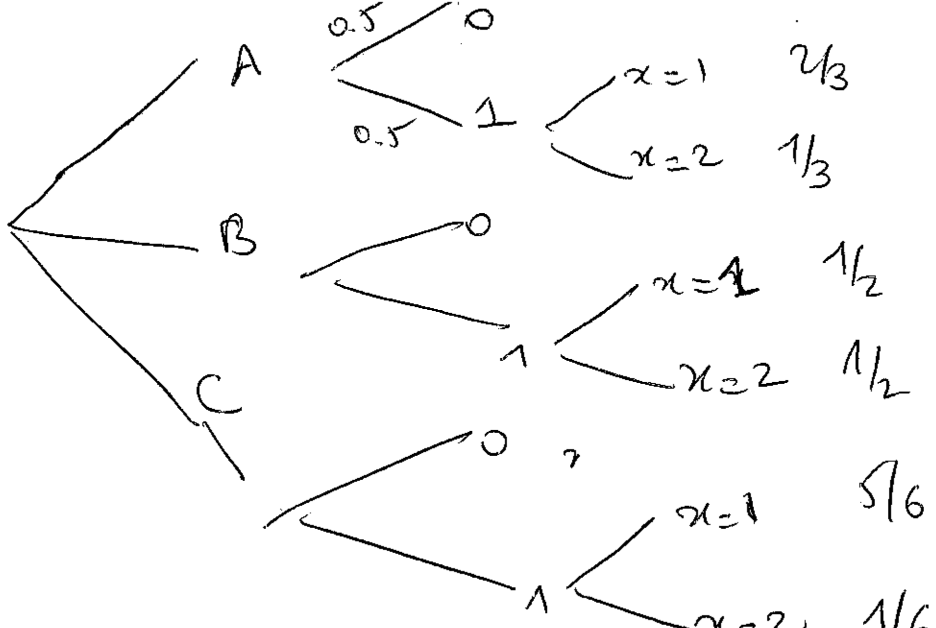
$= 0.737$

(b) $E[X_3 \mid x_1, x_2]$?

$E[X_3 \mid G] = 0.4$ $E[X_3 \mid B] = 0.7$

$E[X_3 \mid x_1, x_2] = (0.4)(0.737) + (0.7)(0.263) = 0.4789$

Ex 6



$$E[X_2|A] = 0\left(\tfrac{1}{2}\right) + 1\left(\tfrac{1}{2}\times\tfrac{2}{3}\right) + 2\left(\tfrac{1}{2}\times\tfrac{1}{3}\right) = \tfrac{2}{3}$$

$$E[X_2|B] = 0\left(\tfrac{1}{2}\right) + 1\left(\tfrac{1}{2}\times\tfrac{1}{2}\right) + 2\left(\tfrac{1}{2}\times\tfrac{1}{2}\right) = \tfrac{3}{4}$$

$$E[X_2|C] = 0\left(\tfrac{1}{2}\right) + 1\left(\tfrac{1}{2}\times\tfrac{5}{6}\right) + 2\left(\tfrac{1}{2}\times\tfrac{1}{6}\right) = \tfrac{7}{12}.$$

$$P(A|X_1=2) = \frac{P(X_1=2|A)P(A)}{P(X_1=2)} = \frac{1/6 \times 1/3}{P(X_1=2)}$$

$$P(X_1=2) = P(X_1=2|A)P(A) + P(X_1=2|B)P(A) + P(X_1=2|C)P(C)$$
$$= (1/6)(1/6) + (1/4)(1/3) + (1/12)(1/3) = 1/6$$

$$\Rightarrow P(A|X_1=2) = \frac{1/18}{1/6} = 1/3.$$

$$P(B|X_1=2) = \frac{1/12}{1/6} = 1/2.$$

$$P(C|X_1=2) = \frac{1/36}{1/6} = 1/6$$

$$E[X_2|X_1=2] = (2/3)(1/3) + (3/4)(1/2) + (7/12)(1/6)$$
$$= 25/36 = 0.6944$$