College of Science.
Department of Statistics & Operations Research

كلية العلوم
قسم الإحصاء وبحوث العمليات

First Midterm Exam
Academic Year 1442-1443 Hijri- SecondSemester

Exam Information معلومات الامتحان			
Course name	تقنيات المعاينة		اسم المقرر
Course Code	ACTU 465		رمز المقرر
Exam Date	2021-02-15	1442-07-03	تاريخ الامتحان
Exam Time	10: 00 AM		وقت الامتحان
Exam Duration	2 hours	ساعتان	مدة الامتحان
Classroom No.			رقم قاعة الاختبار
Instructor Name			اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of 3 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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تعليمات عامة:

- عدد صفحات الامتحان 3 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف و الساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1				
2				
3				
4				
5				
6				
7				
8				

EXAM COVER PAGE

Exercise 1 *Total claims per period* S_i $(i = 1, 2, ..., n)$ *has a compound Poisson distribution. You have determined that* 2670 *claims is necessary for full credibility for total claims per period if the severity distribution is constant. If the severity distribution is lognormal with mean* 1,000 *and variance* 1,500,000*, find the number of claims needed for full credibility of total claims per period.*

$A)$ 6650 $B)$ 6675 $C)$ 6700 $D)$ 6725 $E)$ 6750

Exercise 2 *You are given the following:*
- *The number of claims follows a Poisson distribution.*
- *The variance of the number of claims is* 10*.*
- *The variance of the claim size distribution is* 10*.*
- *The variance of aggregate claim costs is* 500*.*
- *The number of claims and claim sizes are independent.*
- *The full credibility standard has been selected so that actual aggregate claim costs per period will be within* 5% *of expected aggregate claim costs* 95% *of the time.*

Using the methods of limited fluctuation credibility determine the number of claims required for full credibility of aggregate claim costs per period.

Exercise 3 *The aggregate loss in one week,* S_i $(i = 1, 2, ..., n)$*, follows a compound negative binomial distribution, and the severity distribution* Y *is exponential. The full credibility standard has been selected so that actual aggregate claim costs per period will be within* 5% *of expected aggregate losses* 95% *of the time. It is found that the expected number of claims needed for full credibility is* 5,412 *. Suppose that the frequency distribution is modified (but still negative binomial) so that mean* μ_N *and variance* σ_N^2 *of the frequency* N *both increase by* 20%*. Find the full credibility standard for the number of claims needed for the new compound negative binomial distribution (severity is the same exponential distribution as before) and show that full credibility standard is unchanged at* 5,412*.*

Exercise 4 *A company has determined that the limited fluctuation full credibility standard is* 6,600 *claims if:*
(i) The full credibility standard has been selected so that actual aggregate claim costs per period will be within 10% *of expected aggregate claim costs per period* $p\% = 1 - \alpha$ *of the time.*
(ii) The number of claims per period N_i $(i = 1, 2, ..., n)$ *follows a Poisson distribution.*
(iii) Claim sizes Y *follow a lognormal distribution with parameters* μ *(unknown) and* $\sigma^2 = 4$*.*
(iv) The number of claims and claim sizes are independent.
Determine the value of $z_{\alpha/2}$*.*

$A)$ 0.80 $B)$ 0.90 $C)$ 1.00 $D)$ 1.10 $E)$ 1.20

Exercise 5 *Total claims per period* S_i $(i = 1, 2, ..., n)$ *follows a compound Poisson distribution and claim severity* Y *has the pdf*

$$f(y) = \frac{5}{y^6},\ y > 1$$

A full credibility standard based on the number of exposures n *needed has been determined so that the total cost of claims per period* $\bar{S}$ *is within* 5% *of the expected cost* μ_S *with a probability of* 90%. *If the same number of exposures for full credibility of total cost is applied to the number of exposures needed for the frequency variable, the actual number of claims per exposure period* $\bar{N}$ *would be within* $100r$ % *of the expected number of claims per exposure period* μ_N *with probability* 95%.

a) Calculate the coefficient of variation $C_Y = \sigma_Y / \mu_Y$.

b) Find r.

$A)$ 0.054 $\quad B)$ 0.058 $\quad C)$ 0.062 $\quad D)$ 0.066 $\quad E)$ 0.070

A.3.3 One-parameter distributions

A.3.3.1 Exponential—θ

$$
\begin{aligned}
f(x) &= \frac{e^{-x/\theta}}{\theta} \qquad F(x) = 1 - e^{-x/\theta} \\
M(t) &= (1-\theta t)^{-1} \qquad \mathrm{E}[X^k] = \theta^k \Gamma(k+1), \quad k > -1 \\
\mathrm{E}[X^k] &= \theta^k k!, \quad \text{if } k \text{ is an integer} \\
\mathrm{VaR}_p(X) &= -\theta \ln(1-p) \\
\mathrm{TVaR}_p(X) &= -\theta \ln(1-p) + \theta \\
\mathrm{E}[X \wedge x] &= \theta(1 - e^{-x/\theta}) \\
\mathrm{E}[(X \wedge x)^k] &= \theta^k \Gamma(k+1)\Gamma(k+1; x/\theta) + x^k e^{-x/\theta}, \quad k > -1 \\
&= \theta^k k! \Gamma(k+1; x/\theta) + x^k e^{-x/\theta}, \quad k \text{ an integer} \\
\text{mode} &= 0
\end{aligned}
$$

A.3.3.2 Inverse exponential—θ

$$
\begin{aligned}
f(x) &= \frac{\theta e^{-\theta/x}}{x^2} \qquad F(x) = e^{-\theta/x} \\
\mathrm{E}[X^k] &= \theta^k \Gamma(1-k), \quad k < 1 \\
\mathrm{VaR}_p(X) &= \theta(-\ln p)^{-1} \\
\mathrm{E}[(X \wedge x)^k] &= \theta^k G(1-k; \theta/x) + x^k(1 - e^{-\theta/x}), \quad \text{all } k \\
\text{mode} &= \theta/2
\end{aligned}
$$

A.5 Other distributions

A.5.1.1 Lognormal—μ,σ (μ can be negative)

$$
\begin{aligned}
f(x) &= \frac{1}{x\sigma\sqrt{2\pi}} \exp(-z^2/2) = \phi(z)/(\sigma x), \quad z = \frac{\ln x - \mu}{\sigma} \qquad F(x) = \Phi(z) \\
\mathrm{E}[X^k] &= \exp(k\mu + k^2\sigma^2/2) \\
\mathrm{E}[(X \wedge x)^k] &= \exp(k\mu + k^2\sigma^2/2)\Phi\left(\frac{\ln x - \mu - k\sigma^2}{\sigma}\right) + x^k[1 - F(x)] \\
\text{mode} &= \exp(\mu - \sigma^2)
\end{aligned}
$$

Ex 1 $S_1 = \sum^{N_1} Y_i$ ... $S_n = \sum_1^{N_n} Y_i$ **Lognormal**

$N_1 = 2670 = \lambda_0 (1 + C_{Y_1}^2)$

$Y_1 \equiv cte \Rightarrow \sigma_{Y_1}^2 = 0 \to C_{Y_1}^2 = 0$

$\Rightarrow 2670 = \lambda_0$

$N_2 = \lambda_0 (1 + C_{Y_2}^2) = 2670 \left(1 + \frac{1500000}{1000^2}\right)$

$= 6675.$

Ex 2 $\sigma_N^2 = \lambda = 10$; $\sigma_Y^2 = 10$

$V(S) = V\left(\sum_1^N Y_i\right) = \lambda(\mu_Y^2 + \sigma_Y^2) = 500$

$\Rightarrow \mu_Y^2 + \sigma_Y^2 = 50 \to \mu_Y^2 = 40$

$N_f = \lambda_0 (1 + C_Y^2) = \left(\frac{1.96}{0.05}\right)^2 \left(1 + \frac{10}{40}\right)$

$= 1921.$

Ex 3 $S_1 \ldots S_n$

$S = \sum_1^{N_i} Y_i$

$n = \lambda_0 C_S^2 = \lambda_0 \frac{\sigma_S^2}{\mu_S^2}$ $Y \sim \text{Exp}(\text{mean} = \beta)$

$\mu_Y = \beta$ $\sigma_N^2 = \beta^2$

$E(\sum N_i) = n \mu_N = \lambda_0 \mu_N \cdot \sigma_S^2 / \mu_S^2$.

$\mu_S = \mu_N \mu_Y = \beta \mu_N$; $\sigma_S^2 = \sigma_N^2 \mu_Y^2 + \mu_N \sigma_Y^2$

$\Rightarrow E(\sum N_i) = \lambda_0 \mu_N \frac{\beta^2 [\mu_N + \sigma_N^2]}{\beta^2 \mu_N^2} = \beta^2 [\mu_N + \sigma_N^2]$

$= \lambda_0 (1 + \sigma_N^2 / \mu_N) = 5412$

New standard :

$E(\sum N_i) = \lambda_0 \left(1 + \frac{\sigma_N^2 (1+0.2)}{\sigma_N (1+0.2)}\right) = 5412$

Ex 4

$$E(\Sigma N_i) = \lambda_0 (1 + C_y^2) \quad E(N) = 6600$$

$$\mu_y = e^{\mu+2}$$

$$\mu_{y^2} = e^{2\mu+8}$$

$$\sigma_y^2 = e^{2\mu+8} - e^{2\mu+4} = e^{2\mu}(e^8 - e^4)$$

$$C_y^2 = \frac{\sigma_y^2}{\mu_y^2} = \frac{e^{2\mu}(e^8 - e^4)}{e^{2\mu+4}} = e^4 - 1$$

$$\Rightarrow \lambda_0 = \frac{6600}{e^4}$$

$$\lambda_0 = \left(\frac{z}{k}\right)^2 = \left(\frac{z}{0.1}\right)^2 = \frac{6600}{e^4}$$

$$\Rightarrow z = 0.1 \times \sqrt{\frac{6600}{e^4}} = 1.10$$

Ex 5

$$S_1 = \sum_1^{N_1} Y_i, \quad S_2 = \sum_1^{N_2} Y_i \quad \dots \quad S_n = \sum_1^{N_n} Y_i$$

$$P\left\{ -5\% < \frac{\bar{S} - \mu_S}{\mu_S} < 5\% \right\} = 0.90$$

$$\Leftrightarrow P\left\{ -5\% \frac{\mu_S}{\sigma_{\bar{S}}} < \frac{\bar{S} - \mu_S}{\sigma_{\bar{S}}} < 5\% \frac{\mu_S}{\sigma_{\bar{S}}} \right\} = 0.90$$

$$\Rightarrow 0.05 \frac{\mu_S}{\sigma_{\bar{S}}} = 1.645 \quad (z_{\alpha/2})$$

$$\Rightarrow 0.05 \frac{\mu_S}{\sigma_S/\sqrt{n}} = 1.645$$

$$\Rightarrow n_1 = \underbrace{\left(\frac{1.645}{0.05}\right)^2}_{d_0^1} \frac{\sigma_S^2}{\mu_S^2} = \frac{d_0^1}{d}(1 + C_Y^2)$$

$$P\left\{ -r < \frac{\bar{N} - \mu_N}{\mu_N} < r \right\} = 0.95$$

$$\Rightarrow P\left\{ -r \frac{\mu_N}{\sigma_{\bar{N}}} < \frac{\bar{N} - \mu_N}{\sigma_{\bar{N}}} < r \frac{\mu_N}{\sigma_{\bar{N}}} \right\} = 0.95$$

$$\Rightarrow r \frac{\mu_N}{\sigma_{\bar{N}}} = 1.96$$

$$\Rightarrow n_2 = \left(\frac{1.96}{r}\right)^2 \frac{\sigma_N^2}{\mu_N^2} = \left(\frac{1.96}{r}\right)^2 \frac{d}{d^2}$$

$$n_1 = n_2 \Leftrightarrow \left(\frac{1.645}{0.05}\right)^2 \frac{(1 + C_Y^2)}{d} = \left(\frac{1.96}{r}\right)^2 / d$$

$$\Rightarrow r = \left(\frac{0.05 \times 1.96}{1.645}\right) \frac{1}{\sqrt{1 + C_Y^2}} = 0.058$$

$\mu_Y = \int y f(y) = 5/4$

$\mu_{Y^2} = 5/3$

$\sigma_Y^2 = 5/3 - \frac{25}{16} = 5/48$

$C_Y^2 = 2/30$