كلية العلوم
قسم الإحصاء وبحوث العمليات

College of Science.
Department of Statistics & Operations Research

First Midterm Exam
Academic Year 1442-1443 Hijri- First Semester

Exam Information معلومات الامتحان			
Course name	Credibility		اسم المقرر
Course Code	Actu 465		رمز المقرر
Exam Date	2021-10-25	1442-03-15	تاريخ الامتحان
Exam Time	10: 00 AM		وقت الامتحان
Exam Duration	2 hours	ساعتان	مدة الامتحان
Classroom No.			رقم قاعة الاختبار
Instructor Name			اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of 1 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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تعليمات عامة:

- عدد صفحات الامتحان 1 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف و الساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1				
2				
3				
4				
5				
6				
7				
8				

EXAM COVER PAGE

Exercise 1 *Total claims per period* S_i $(i = 1, 2, ..., n)$ *has a compound Poisson distribution. You have determined that a sample size of* 2670 *claims is necessary for full credibility for total claims per period if the severity distribution is constant. If the severity distribution is lognormal with mean* 1000 *and variance* 1,500,000*, find the number of claims needed for full credibility of total claims per period.*

Exercise 2 *A compound distribution* S_i $(i = 1, 2, ..., n)$ *has a Poisson frequency distribution* N *with mean* λ*. For parts (a) and (b), assume that the severity distribution* Y *is uniform on the interval* $[0, \theta]$*.*
(a) Limited fluctuation credibility is applied to Y *based on the sample mean of* Y *being within* 5% *of the true mean of* Y *with probability* 90%*. Find expressions for*
(i) the number of observations n *of* Y *needed for full credibility, and*
(ii) the expected sum of the observed values of Y *needed for full credibility.*
(b) Limited fluctuation readability is applied to S *based on the sample mean of* S *being within* 5% *of the true mean of* S *with probability* 90%*. Find expressions for*
(i) the expected number of observations of S *needed for full credibility, and*
(ii) the expected sum of the observed values of S *needed for full credibility, and*
(iii) the expected total number of claims needed for full credibility.

Exercise 3 *Aggregate claims per period* S_i $(i = 1, 2, ..., n)$ *has a compound Poisson distribution. You have determined that a sample size of* 4,000 *claims is necessary for full credibility for aggregate claims per period if the severity distribution is constant. If the severity distribution is exponential with mean* 1,000*, find the number of claims needed for full credibility of aggregate claims per period.*

Exercise 4 *A company has determined that the limited fluctuation full credibility standard is* 2,000 *claims if:*
(i) The total number of claims is to be within 3% *of the true value with probability* p*.*
(ii) The number of claims follows a Poisson distribution.
The standard is changed so that the total cost of claims is to be within 5% *of the true value with probability* p*, where claim severity has probability density function:*

$$f(x) = \frac{1}{10,000},\ 0 \leq x \leq 10,000$$

Using limited fluctuation credibility, calculate the expected number of claims necessary to obtain full credibility under the new standard.

Ex 1 $S_1, \dots, S_n$

2) FC $\Leftrightarrow$ $n \geq \lambda_0 C_S^2 = \frac{\lambda_0}{\lambda}(1 + C_Y^2)$

$= \lambda_0 / \lambda$

$E(\sum_1^n N_i) = n\lambda \geq \lambda_0$

$\Rightarrow 2670 = \lambda_0$

(3) $EY = 1000$ $\sigma_Y^2 = 1500{,}000$

$E(\sum_1^n N_i) = \lambda_0(1 + C_Y^2)$

$= 2670\left(1 + \frac{1{,}5}{1}\right) = 6675$

Ex 3 $E(\sum N_i)_1 = \lambda_0(1 + C_Y^2) = \lambda_0 = 4000$

$E(\sum N_i)_2 = \lambda_0(1 + C_Y^2) = 4000\left(1 + \frac{1000^2}{1000^2}\right)$

$= 8000$

Ex 4 $P\left(\frac{N - \mu_N}{\mu_N} < k\right)$

$k\frac{\mu_N}{\sigma_N} \geq z_0 \Leftrightarrow k\sqrt{\lambda} \geq z_0$

$\Leftrightarrow \lambda_N \geq \lambda_0^1$

$\lambda_0^1 = \left(\frac{z}{3\%}\right)^2 = 2000 \Rightarrow z^2 = 2000 \times 3\%^2$

$E(\sum_1^{n=1} N_i) = m\lambda = \lambda_0^2(1 + C_Y^2) = \left(\frac{z}{5\%}\right)^2\left(1 + \frac{10{,}000^2/12}{5{,}000^2}\right)$

Ex 2

$y_1 \ldots, y_n$

$$n \geq \lambda_0 C_Y^2 = \lambda_0 \frac{\theta^2/12}{\theta^2/4} = \frac{\lambda_0}{3}$$

$$= \left(\frac{1.645}{0.05}\right)^2 / 3$$

$$= 360.80$$

$$E\left(\sum_1^n Y_i\right) = n\mu_Y$$

$$= n_\beta \theta/2$$

$$= 180.4 \times \theta$$

$S_1 \ldots S_n$

$$n \geq \frac{\lambda_0}{\lambda}(1 + C_Y^2) = \frac{\left(\frac{1.645}{0.05}\right)^2}{\lambda}\left(1 + \frac{1}{3}\right)$$

$$= \frac{1443}{\lambda}$$

$$E\left(\sum_1^n S_i\right) = n\mu_S = n\lambda\, \theta/2$$

$$= \frac{1443}{2} \times \theta$$

$$= 721 \times \theta$$

$$E\left(\sum_1^n N_i\right) = n\mu_N = n\lambda \geq 1443$$