جامعة الملك سعود
King Saud University

College of Science.
Department of Statistics & Operations Research

كلية العلوم
قسم الإحصاء وبحوث العمليات

Second Midterm Exam
Academic Year 1443-1444 Hijri- First Semester

Exam Information معلومات الامتحان			
Course name	**Credibility**		اسم المقرر
Course Code	**Actu 465**		رمز المقرر
Exam Date	2021-11-08	1443-04-03	تاريخ الامتحان
Exam Time	**10: 00 AM**		وقت الامتحان
Exam Duration	2 hours	ساعتان	مدة الامتحان
Classroom No.			رقم قاعة الاختبار
Instructor Name			اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of 2 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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تعليمات عامة:

- عدد صفحات الامتحان 2 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف و الساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1				
2				
3				
4				
5				
6				
7				
8				

EXAM COVER PAGE

Exercise 1 *The Slippery Rock Insurance Company is reviewing its rates. It wants the expected number of claims required for full credibility to be based on a probability level of* 90% *and a range parameter of* 5%. *It estimates that individual claim losses* Y *are mutually independent and identically distributed according to the probability density function*

$$f(x) = \frac{1}{200,000}, \text{for } 0 < x < 200,000.$$

Assume that the number of claims follows a Poisson distribution with mean λ.
(a) Show that the variance of aggregate claims, $Var(S)$, *is equal to* $\lambda(200,000)^2(\frac{1}{3})$.
(b) Assuming that the most recent period of observation contains 1082 *claims, show that the credibility factor for that period is* $Z = 0.866$.

Exercise 2 *Assume there are two different types of drivers, good* (G) *and bad drivers* (B). *The variable* X *is the number of claims in any one year.*

x	$\Pr(x\,\vert G)$	$\Pr(x\,\vert B)$	
0	0.7	0.5	$P(G) = 0.75$
1	0.2	0.3	$P(B) = 0.25$
2	0.1	0.2	

Suppose a policyholder had 0 *claims the first year and* 1 *claim the second year.*
Determine
a) The posterior probability $\pi(G\,|x_1, x_2)$
b) Determine the Bayesian estimate of this insured's claim count in the next (third) policy year.

0.279 0.379 0.479 0.579

Exercise 3 *A risk class is made up of three equally sized groups of individuals. Groups are classified as Type A, Type B and Type C. Any individual of any type has probability of* 0.5 *of having no claim in the coming year and has a probability of* 0.5 *of having exactly* 1*. Each claim is for amount* 1 *or* 2 *when a claim occurs. Suppose that the claim distributions given that a claim occurs, for the three types of individuals are*

$$\Pr(\textit{claim of amount } x \,|\, \textit{Type } A \textit{ and a claim occurs}) = \begin{cases} 2/3 & x = 1 \\ 1/3 & x = 2 \end{cases}$$

$$\Pr(\textit{claim of amount } x \,|\, \textit{Type } B \textit{ and a claim occurs}) = \begin{cases} 1/2 & x = 1 \\ 1/2 & x = 2 \end{cases}$$

$$\Pr(\textit{claim of amount } x \,|\, \textit{Type } C \textit{ and a claim occurs}) = \begin{cases} 5/6 & x = 1 \\ 1/6 & x = 2 \end{cases}$$

An insured is chosen at random from the risk class and is found to have a claim of amount 2 *in Year 1. Determine the Bayesian estimate of this insured's claim amount in the next policy year.*

$\frac{25}{36}$ $\frac{3}{4}$ $\frac{29}{36}$ $\frac{11}{12}$

465 Mid2 Ex

Exs ⑤ $Y \sim U[0, 200{,}000]$

(a) $\mu_Y = 100{,}000$; $\sigma_Y^2 = \frac{200{,}000^2}{12}$

② $Vars = \lambda(\mu_Y^2 + \sigma_Y^2) = \lambda\left(\left(\frac{200{,}000}{2}\right)^2 + \frac{200{,}000^2}{12}\right)$

$= \lambda\,(200{,}000)^2\left(\underbrace{1/4 + 1/12}_{=1/3}\right)$

(b) $\Sigma N_i = 1082$

$(\Sigma N_i)_f = \lambda_0(1 + C_Y^2) = \lambda_0 \frac{\sigma_S^2}{\mu_S^2}$

③ $= \left(\frac{1.645}{0.05}\right)^2 (2)$ ~~+~~

$= \left(\frac{1.645}{0.05}\right)^2\left(1 + \frac{200{,}000^2/12}{100{,}000^2}\right)$

$= \left(\frac{1.645}{0.05}\right)^2\left(1 + \frac{4}{12}\right) = \left(\frac{1.645}{0.05}\right)^2 (4/3)$

$= 1443.213$

$z = \sqrt{\frac{\Sigma N_i}{(\Sigma N_i)_f}} = \sqrt{\frac{1082}{1443}} = 0.8658$

Ex2 ⑧ $Pr(G \mid x_1, x_2) = \frac{P(x_1=0, x_2=1 \mid G)\,\pi(G)}{P(x_1=0, x_2=1 \mid G)\,\pi(G) + P(\quad \mid B)\,\pi(B)}$

② $= \frac{(0.7)(0.2)(0.75)}{(0.7)(0.2)(0.75) + (0.5)(0.3)(0.25)} = 0.737 = 14/19$

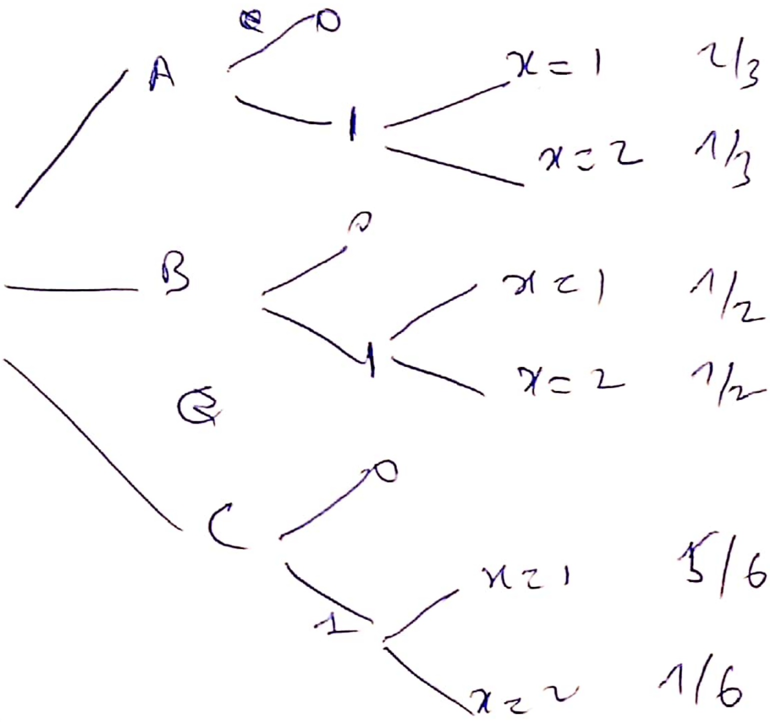
② $E(X_3 \mid G) = 0.4$ ② $E(X_3 \mid B) = 0.7$

$\rightarrow E(X_3 \mid x_1, x_2) = (0.4)(0.737) + (0.7)(0.263) = 0.4789$

②

Scanné avec CamScanner

Ex3 (12)



1 $E[X_2|A] = 0(1/2) + 1(1/2 \times 2/3) + 2(1/2 \times 1/3) = 2/3$

1 $E[X|B] = 0(1/2) + 1(1/2 \times 1/2) + 2(1/2 \times 1/2) = 3/4$

1 $E[X|C] = 0(1/2) + 1(1/2 \times 5/6) + 2(1/2 \times 1/6) = 7/12$

$$P(A|X_1 = 2) = \frac{P(x_1 = 2|A)\,\pi(A)}{\underbrace{P(x_1=2|A)\,\pi(A) + P(x_1=2|B)\,\pi(B) + P(x_1=2|C)\,\pi(C)}_{D}}$$

1 $D = (1/6)(1/3) + (1/4)(1/3) + (1/12)(1/3) = 1/6$

2 $P(A|x_1=2) = \frac{1/6 \times 1/3}{1/6} = 1/3$

2 $P(B|x_1=2) = \frac{1/4 \times 1/3}{1/6} = 1/2$

2 $P(C|x_1=2) = \frac{1/12 \times 1/3}{1/6} = 1/6$

2 $E[X_2|x_1=2] = 2/3(1/3) + 3/4 \times 1/2 + \frac{7}{12} \times 1/6 = \frac{25}{36}$

$= 0{,}6944$

20