Exercise 1 *In a portfolio of insurance policies, the number of claims for each policyholder in each year, denoted by* N*, may be* $0, 1,$ *or* 2*, with the following pf:* $f_N(0) = 0.1, f_N(1) = 0.9 - \theta,$*and* $f_N(2) = \theta$*. The prior pdf of* Θ *is*

$$f_\Theta(\theta) = \frac{\theta^2}{0.039} \qquad 0.2 < \theta < 0.5$$

A randomly selected policyholder has two claims in Year 1 and two claims in Year 2. Determine the Bayes estimate of the expected number of claims in Year 3 of this policyholder.
(1) 1.722 (2) 0.722 (3) 0.322 (4) 1.319

Exercise 2 *Suppose that* X *and* Y *have joint distribution*

$$f_{X,Y}(x, y) = \begin{cases} 8xy & 0 < x < y < 1 \\ 0 & otherwise \end{cases}$$

Find $E(X\,|Y\,)$*and* $E(Y\,|X\,)$*.*

Exercise 3 *The number of stops* X *in a day for a delivery truck driver is Poisson with mean* λ*. Conditional on their being* $X = x$ *stops, the expected distance driven by the driver* Y *is Normal with a mean of* αx *miles, and a standard deviation of* βx *miles. Give the mean and variance of the numbers of miles she drives per day.*

Credibility
HW1

Ex1

$$N|_\theta \begin{cases} 0 & 0.1 \\ 1 & 0.9-\theta \\ 2 & \theta \end{cases}$$

$n_1 = n_2 = 2$; $n = (2,2)$

$$E(N_3|\theta) = 1(0.9-\theta)+2\theta = 0.9+\theta$$

$$\pi(\theta|N=n) = \frac{f(n|\theta)\pi(\theta)}{\int f(n|\theta)\pi(\theta)d\theta} = \frac{\theta^2\left(\frac{\theta^2}{0.039}\right)}{\int f(n|\theta)\pi(\theta)d\theta} = \frac{\theta^4}{C}$$

$$C = \int_{0.2}^{0.5} \theta^4 \, d\theta = \left[\frac{\theta^5}{5}\right]_{0.2}^{0.5} = \frac{1}{5}\left[0.5^5 - 0.2^5\right]$$

$$E(N_3|N=n) = \int E(N_3|\theta)\,\pi(\theta|N=n)\,d\theta$$

$$= \int (0.9+\theta)\frac{\theta^4}{C}\,d\theta = \frac{1}{C}\left[\int 0.9\theta^4 d\theta + \int \theta^5 d\theta\right]$$

$$= \frac{1}{C}\left[0.9C + \frac{1}{6}\left(0.5^6 - 0.2^6\right)\right] = \cdots = 1.3193$$

Ex2

$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$$

$0<y<1$,

$$f_Y(y) = \int_0^y f(x,y)\,dx = \int_0^y 8xy\,dx = 8y\left[\frac{x^2}{2}\right]_0^y = 4y^3$$

$$\Rightarrow f_{X|Y}(x|y) = \frac{8xy}{4y^3} = 2\frac{x}{y^2}; \quad 0<x<y<1.$$

$$E(X|Y=y) = \int_0^y x\left(\frac{2x}{y^2}\right)dx = \frac{2}{y^2}\left(\frac{y^3}{3}\right) = \frac{2y}{3}; \quad 0<y<1$$

$$f_{Y|X}(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)}$$

$0<x<1$,

$$f_X(x) = \int_x^1 8xy\,dy = 8x\left[\frac{y^2}{2}\right]_x^1 = 4x(1-x^2)$$

$$\Rightarrow f_{Y|X}(y|x) = \frac{8xy}{4x(1-x^2)} = \frac{2y}{1-x^2}, \quad 0<x<y<1$$

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$$E(Y|X=x) = \int_x^1 y\left(\frac{2y}{1-x^2}\right)dy = \frac{2}{1-x^2}\left[\frac{y^3}{3}\right]_x^1$$

$$= \frac{2}{3(1-x^2)}(1-x^3), \quad 0 < x < 1$$

Ex 3 $\quad X \sim \mathcal{P}(\lambda); \quad Y|_{X=x} \sim \mathcal{N}(\text{mean} = \alpha x, \ \sigma = \beta x)$

$$\Rightarrow \begin{cases} E(Y|X) = \alpha X \\ V(Y|X) = \beta^2 X^2 \end{cases}$$

$$E(Y) = E(E(Y|X)) = E(\alpha X) = \alpha\lambda$$

$$V(Y) = V(E(Y|X)) + E(V(Y|X))$$

$$= V(\alpha X) + E(\beta^2 X^2)$$

$$= \alpha^2\lambda + \beta^2\left[(EX)^2 + VX\right]$$

$$= \alpha^2\lambda + \beta^2(\lambda^2 + \lambda)$$