Exercise 1 *For a portfolio of insurance policies the annual claim amount* X *of a policy has the following pdf*

$$f_X(x\,|\theta) = \frac{2x}{\theta^2}, \qquad 0 < x < \theta$$

The prior distribution of Θ *has the following pdf*

$$f_\Theta(\theta) = 4\theta^3, \qquad 0 < \theta < 1$$

A randomly selected policy has claim amount 0.1 *inYear 1. Determine the Bühlmann credibility estimate of the expected claim amount of the selected policy in Year 2.*

Exercise 2 *You are given the following information about workers compensation coverage:*
(i) The number of claims for an employee during the year follows a Poisson distribution with mean $(100 - p)/100$*; where* p *is the salary (in thousands) for the employee.*
ii) The distribution of p *is uniform on the interval* $(0; 100]$*. An employee is selected at random. During the last* 4 *years, the employee has had a total of* 5 *claims.*
Determine the Buhlmann credibility estimate for the expected number of claims the employee will have next year.

Credibility
HW 2

Ex 1

$$f(x|\theta) = \frac{2x}{\theta^2}, \quad 0 < x < \theta$$

$$\pi(\theta) = 4\theta^3, \quad 0 < \theta < 1$$

$x_1 = 0.1$.

$$\mu(\theta) = E(X|\theta) = \int_0^\theta \frac{2x^2}{\theta^2}\,dx = \frac{2}{\theta^2}\left[\frac{x^3}{3}\right]_0^\theta = \frac{2\theta}{3}.$$

$$E(X^2|\theta) = \int_0^\theta \frac{2x^3}{\theta^2}\,dx = \frac{2}{\theta^2}\left[\frac{x^4}{4}\right]_0^\theta = \frac{\theta^2}{2}$$

$$v(\theta) = V(X|\theta) = \frac{\theta^2}{2} - \left(\frac{2\theta}{3}\right)^2 = \frac{\theta^2}{18}$$

$$\mu = E(\mu(\theta)) = \int_0 \frac{8}{3}(\theta^4\,d\theta) = \cdots = \frac{8}{15}$$

$$v = E(v(\theta)) = \frac{1}{18}\int_0^1 4\theta^5\,d\theta = \cdots = \frac{1}{27}$$

$$E(\mu^2(\theta)) = \left(\frac{2}{3}\right)^2 \int_0^1 4\theta^5\,d\theta = \frac{4}{9}\cdot\left(\frac{4}{6}\right) = \frac{8}{27}$$

$$a = V(\mu(\theta)) = \frac{8}{27} - \left(\frac{8}{15}\right)^2 = \frac{8}{675}.$$

$$k = \frac{v}{a} = \frac{675}{8} \times \frac{1}{27} = \frac{25}{8}$$

$$Z = \frac{1}{1+k} = \frac{1}{1+25/8} = \frac{8}{33}.$$

$$BC = Z(0.1) + (1-Z)\mu$$

$$= \frac{8}{33}(0.1) + \frac{25}{33}\left(\frac{8}{15}\right) \approx 0.428$$

Ex 2

$$X \sim \mathcal{P}\left(\lambda = \frac{100-p}{100}\right)$$

$$P \sim \mathcal{U}_{(0,100)}.$$

$$E(X|p) = V(X|p) = \frac{100-p}{100}.$$

$$\sum_{i=1}^{4} x_i = 5 \Rightarrow \bar{x} = 5/4.$$

$$\mu = E(\mu(p)) = E(E(X|p)) = E\left(\frac{100-p}{100}\right)$$

$$= \frac{1}{100}(100 - \underbrace{Ep}_{50}) = 1/2.$$

$$v = E(V(X|p)) = E\left(\frac{100-p}{100}\right) = 1/2.$$

$$a = V(\mu(p)) = V\left(\frac{100-p}{100}\right) = \frac{1}{100^2} V(p) = \frac{1}{100^2} \frac{p^2}{12}$$

$$= 1/12.$$

$$k = \frac{v}{a} = 6.$$

$$Z = \frac{n}{n+k} = \frac{4}{4+6} = 4/10.$$

$$BC = Z\bar{x} + (1-Z)\mu$$

$$= \frac{4}{10}\left(\frac{5}{4}\right) + \frac{6}{10}(1/2) = \frac{5}{10} + \frac{3}{10} = 0.8$$