EXAMPLE 4.1.7 Let $f(x) = x^2 + \frac{2}{x}$. Find the absolute extrema of f on $\left[\frac{1}{4}, 2\right]$.

Solution We follow the four steps in the procedure to find the extrema of f.

1. Find the critical numbers by differentiating $f(x)$. So,

$$f'(x) = 2x - \frac{2}{x^2} = \frac{2x^3 - 2}{x^2}$$

Therefore, $f'(x) = 0$ means that

$$2x^3 - 2 = 0$$
$$x^3 = 1$$
$$x = 1 \in \left(\frac{1}{4}, 2\right),$$

and the derivative $f'(x)$ does not exist at $x = 0$; however, the function is not defined at $x = 0$. So, f has only $x = 1$ as a critical number.

2. Find the function value at $x = 1$,

$$f(1) = 3$$

3. Find the function values at the endpoints $x = \frac{1}{4}$ and $x = 2$,

$$f\left(\frac{1}{4}\right) = \frac{129}{16} = 8.0625 \quad \text{and} \quad f(2) = 5$$

4. Finally, f has the maximum value $f\left(\frac{1}{4}\right) = 8.0625$ and the minimum value $f(1) = 3$, as illustrated in Figure 4.1.13.

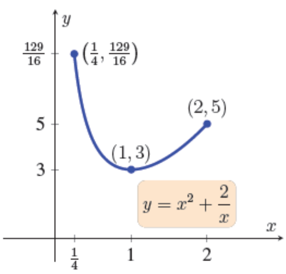

Figure 4.1.13