College of Science.
Department of Statistics & Operations Research

كلية العلوم
قسم الإحصاء وبحوث العمليات

Final Exam
Academic Year 1443-1444 Hijri- First Semester

Exam Information معلومات الامتحان			
Course name	**Loss**		اسم المقرر
Course Code	**Actu 466**		رمز المقرر
Exam Date	2021-12-30	1442-05-26	تاريخ الامتحان
Exam Time	**08: 00 AM**		وقت الامتحان
Exam Duration	2 hours	ساعتان	مدة الامتحان
Classroom No.			رقم قاعة الاختبار
Instructor Name			اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of 1 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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تعليمات عامة:

- عدد صفحات الامتحان 1 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف و الساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة
This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1				
2				
3				
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6				
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8				

EXAM COVER PAGE

Exercise 1 *You are given that losses follow a Pareto distribution with* $\alpha = 3$ *and* $\theta = 1,000$*. The company implements a franchise deductible so that the* $E(Y^P)$ *with the franchise deductible is* 130% *of* $E(X)$ *without any deductible. Calculate the franchise deductible.*

Exercise 2 *Losses follow a uniform distribution over the range of* 0 *to* 1000*. Calculate the Loss Elimination Ratio if an ordinary deductible of* 200 *is applied.*

Exercise 3 *Under an unmodified geometric distribution,* $Var(N) = 20$*. Under a zero-modified geometric distribution,* $Var(N) = 20.25$*. The parameter* β *is the same for both distributions. Calculate* p_0^M*.*

B.2.1.2 Geometric—β

$$
\begin{aligned}
p_0 &= \frac{1}{1+\beta}, \quad a = \frac{\beta}{1+\beta}, \quad b = 0 \qquad p_k = \frac{\beta^k}{(1+\beta)^{k+1}} \\
\mathrm{E}[N] &= \beta, \quad \mathrm{Var}[N] = \beta(1+\beta) \qquad P(z) = [1-\beta(z-1)]^{-1}.
\end{aligned}
$$

This is a special case of the negative binomial with $r = 1$.

B.2.1.3 Binomial—$q, m, (0 < q < 1,\ m$ an integer$)$

$$
\begin{aligned}
p_0 &= (1-q)^m, \quad a = -\frac{q}{1-q}, \quad b = \frac{(m+1)q}{1-q} \\
p_k &= \binom{m}{k} q^k (1-q)^{m-k}, \quad k = 0, 1, \ldots, m \\
\mathrm{E}[N] &= mq, \quad \mathrm{Var}[N] = mq(1-q) \qquad P(z) = [1+q(z-1)]^m.
\end{aligned}
$$

B.2.1.4 Negative binomial—β, r

$$
\begin{aligned}
p_0 &= (1+\beta)^{-r}, \quad a = \frac{\beta}{1+\beta}, \quad b = \frac{(r-1)\beta}{1+\beta} \\
p_k &= \frac{r(r+1)\cdots(r+k-1)\beta^k}{k!(1+\beta)^{r+k}} \\
\mathrm{E}[N] &= r\beta, \quad \mathrm{Var}[N] = r\beta(1+\beta) \qquad P(z) = [1-\beta(z-1)]^{-r}.
\end{aligned}
$$

B.3 The $(a, b, 1)$ class

To distinguish this class from the $(a, b, 0)$ class, the probabilities are denoted $\Pr(N = k) = p_k^M$ or $\Pr(N = k) = p_k^T$ depending on which subclass is being represented. For this class, p_0^M is arbitrary (that is, it is a parameter) and then p_1^M or p_1^T is a specified function of the parameters a and b. Subsequent probabilities are obtained recursively as in the $(a, b, 0)$ class: $p_k^M = (a + b/k)p_{k-1}^M$, $k = 2, 3, \ldots$, with the same recursion for p_k^T There are two sub-classes of this class. When discussing their members, we often refer to the "corresponding" member of the $(a, b, 0)$ class. This refers to the member of that class with the same values for a and b. The notation p_k will continue to be used for probabilities for the corresponding $(a, b, 0)$ distribution.

B.3.1 The zero-truncated subclass

The members of this class have $p_0^T = 0$ and therefore it need not be estimated. These distributions should only be used when a value of zero is impossible. The first factorial moment is $\mu_{(1)} = (a+b)/[(1-a)(1-p_0)]$, where p_0 is the value for the corresponding member of the $(a, b, 0)$ class. For the logarithmic distribution (which has no corresponding member), $\mu_{(1)} = \beta/\ln(1+\beta)$. Higher factorial moments are obtained recursively with the same formula as with the $(a, b, 0)$ class. The variance is $(a+b)[1-(a+b+1)p_0]/[(1-a)(1-p_0)]^2$.For those members of the subclass which have corresponding $(a, b, 0)$ distributions, $p_k^T = p_k/(1-p_0)$.

A.2.3 Two-parameter distributions

A.2.3.1 Pareto (Pareto Type II, Lomax)—α, θ

$$
\begin{aligned}
f(x) &= \frac{\alpha\theta^\alpha}{(x+\theta)^{\alpha+1}} \qquad F(x) = 1 - \left(\frac{\theta}{x+\theta}\right)^\alpha \\
\mathrm{E}[X^k] &= \frac{\theta^k\Gamma(k+1)\Gamma(\alpha-k)}{\Gamma(\alpha)}, \quad -1 < k < \alpha \\
\mathrm{E}[X^k] &= \frac{\theta^k k!}{(\alpha-1)\cdots(\alpha-k)}, \quad \text{if } k \text{ is an integer} \\
\mathrm{VaR}_p(X) &= \theta[(1-p)^{-1/\alpha} - 1] \\
\mathrm{TVaR}_p(X) &= \mathrm{VaR}_p(X) + \frac{\theta(1-p)^{-1/\alpha}}{\alpha-1}, \quad \alpha > 1 \\
\mathrm{E}[X \wedge x] &= \frac{\theta}{\alpha-1}\left[1 - \left(\frac{\theta}{x+\theta}\right)^{\alpha-1}\right], \quad \alpha \neq 1 \\
\mathrm{E}[X \wedge x] &= -\theta \ln\left(\frac{\theta}{x+\theta}\right), \quad \alpha = 1 \\
\mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k\Gamma(k+1)\Gamma(\alpha-k)}{\Gamma(\alpha)}\beta[k+1, \alpha-k; x/(x+\theta)] + x^k\left(\frac{\theta}{x+\theta}\right)^\alpha, \quad \text{all } k \\
\text{mode} &= 0
\end{aligned}
$$

A.2.3.2 Inverse Pareto—τ, θ

$$
\begin{aligned}
f(x) &= \frac{\tau\theta x^{\tau-1}}{(x+\theta)^{\tau+1}} \qquad F(x) = \left(\frac{x}{x+\theta}\right)^\tau \\
\mathrm{E}[X^k] &= \frac{\theta^k\Gamma(\tau+k)\Gamma(1-k)}{\Gamma(\tau)}, \quad -\tau < k < 1 \\
\mathrm{E}[X^k] &= \frac{\theta^k(-k)!}{(\tau-1)\cdots(\tau+k)}, \quad \text{if } k \text{ is a negative integer} \\
\mathrm{VaR}_p(X) &= \theta[p^{-1/\tau} - 1]^{-1} \\
\mathrm{E}[(X \wedge x)^k] &= \theta^k\tau\int_0^{x/(x+\theta)} y^{\tau+k-1}(1-y)^{-k}dy + x^k\left[1 - \left(\frac{x}{x+\theta}\right)^\tau\right], \quad k > -\tau \\
\text{mode} &= \theta\frac{\tau-1}{2}, \quad \tau > 1, \text{ else } 0
\end{aligned}
$$

A.2.3.3 Loglogistic (Fisk)—γ, θ

$$
\begin{aligned}
f(x) &= \frac{\gamma(x/\theta)^\gamma}{x[1+(x/\theta)^\gamma]^2} \qquad F(x) = u, \quad u = \frac{(x/\theta)^\gamma}{1+(x/\theta)^\gamma} \\
\mathrm{E}[X^k] &= \theta^k\Gamma(1+k/\gamma)\Gamma(1-k/\gamma), \quad -\gamma < k < \gamma \\
\mathrm{VaR}_p(X) &= \theta(p^{-1} - 1)^{-1/\gamma} \\
\mathrm{E}[(X \wedge x)^k] &= \theta^k\Gamma(1+k/\gamma)\Gamma(1-k/\gamma)\beta(1+k/\gamma, 1-k/\gamma; u) + x^k(1-u), \quad k > -\gamma \\
\text{mode} &= \theta\left(\frac{\gamma-1}{\gamma+1}\right)^{1/\gamma}, \quad \gamma > 1, \text{ else } 0
\end{aligned}
$$

loss ~~...~~ Final

Ex 1 $X \sim Pareto(\alpha=3, \theta=1000)$

$$E(Y^P) = \frac{E(X) - E(X \wedge d)}{P(X>d)} + d \qquad E(X) = \frac{\theta}{\alpha-1} = 500$$

$$= \frac{\frac{\theta}{\alpha-1}\left(\frac{\theta}{d+\theta}\right)^{\alpha-1}}{\left(\frac{\theta}{d+\theta}\right)^{\alpha}} + d = \frac{\frac{\theta}{\alpha-1}}{\left(\frac{\theta}{d+\theta}\right)} + d = \frac{d+\theta}{\alpha-1} + d$$

$$= \frac{d+1000}{2} + d = \frac{3d+1000}{2} = 1.3\,(500)$$

$\Leftrightarrow$ $3d + 1000 = 1.3 \times 1000 = 1300$

$\Leftrightarrow$ $3d = 300 \Rightarrow \boxed{d = 100}$

Ex 2 $X \sim U(0, w)$ $\quad f(x) = 1/w$, $w = 1000$

$$LER = \frac{E(X) - E(Y^L)}{E(X)} = \frac{E(X \wedge d)}{E(X)}, \quad d = 200$$

$$X \wedge d = \begin{cases} X & x \le d \\ d & x > d \end{cases}$$

$E(X) = w/2 = 500.$

$$E(X \wedge d) = \int_0^{200} \frac{x\,dx}{1000} + 200\left[\frac{1000-200}{1000}\right]$$

$$= \frac{1}{1000}\left[\frac{200^2}{2}\right] + 200\,[8/10] = 180$$

$$LER = \frac{180}{500} = \boxed{0.36}$$

Ex 3 $N \sim Geom.(\beta)$. $V(N) = 20$ $\quad p_0 = (1+\beta)^{-1}$

$N^{(M)}$ zero modif $\Rightarrow$ $p_k^{(M)} = \frac{1-p_0^M}{1-p_0} p_k.$

$V(N) = \beta(1+\beta) = 20 \Rightarrow \beta = 4.$

$$E(N^{(M)}) = \sum_{n}^{\infty} k\, p_k^{(M)} = \frac{1-p_0^M}{1-p_0} E(N) = a\,E(N) = a\beta = 4a$$

$$E(N^{(M)2}) = \frac{1-p_0^M}{1-p_0} E(N^2) = a\,E(N^2)$$

$$V(N^{(M)}) = a\,\underbrace{E(N^2)}_{E(N)^2+V(N)} - (4a)^2 = a\left[4^2+20\right] - 16a^2$$

$$= 36a - 16a^2 = 20.25$$

$\Leftrightarrow$ $16a^2 - 36a + 20.25 = 0 \Rightarrow (4a - 4.5)^2 = 0$

$$\Rightarrow a = \frac{4.5}{4} = \frac{1-p_0^M}{1-p_0} \Rightarrow p_0^M = 1 - a(1-p_0)$$

$$= 1 - \frac{4.5}{4}\left(1 - \frac{1}{5}\right) = \boxed{0.1}$$

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