King Saud University	College of Sciences	Department of Mathematics	
First Examination	Math 132	Semester I (1443)	Time:1H30

Exercise 1: (12 pts)

1. Decide whether the following propositions is a tautology or a contradiction or a contingency?

$$(p \vee \neg q) \rightarrow (r \wedge \neg p). \quad \text{(3 pts)}$$

2. Without using truth tables, prove that the following conditional statement is a Tautology:

$$[(p \vee q) \wedge \neg p] \rightarrow q. \quad \text{(3 pts)}$$

3. Prove that the following conditional statement is a Contradiction:

$$[p \wedge (p \rightarrow q)] \wedge (p \rightarrow \neg q). \quad \text{(2 pts)}$$

4. Without using truth tables, prove the following logical equivalence:

$$[(\neg p \rightarrow q) \vee (\neg q \rightarrow r)] \equiv p \vee q \vee r. \quad \text{(2 pts)}$$

5. Determine the truth value of each of the following statements if the domain consists of all real numbers. (Justify your answer)
   (a) $\forall x \in \mathbb{R};\ (x^2 < x^3)$. (1 pts)
   (b) $\exists x \in \mathbb{R};\ (-x)^3 = -x^3$. (1 pts).

Exercise 2: (13 pts)

1. Let n be an integer. Prove that: $n+3$ is odd if and only if $3n+4$ is even. (3 pts)

2. Let x, y and z be three real numbers. Prove by contradiction that:
   if $(2x^3 + 3y^4 + 5z^2 = 144)$ then, $(x \leq 2$ or $y \leq 2$ or $z \leq 4)$. (2 pts)

3. Use mathematical induction to prove the following statement:

$$2 + 4 + 6 + \cdots + 2n = n(n+1), \quad \text{for each integer } n, \text{ with } n \geq 1. \quad \text{(4 pts)}$$

4. Consider the sequence $\{u_n\}_{n=0}^{\infty}$ defined as follows: $\begin{cases} u_0 = 0 \\ u_1 = 2 \\ u_{n+1} = 4u_n - 3u_{n-1}; \quad n \geq 1 \end{cases}$

   Use mathematical induction to prove the following statement:

$$u_n = 3^n - 1, \quad \text{for each integer } n, \text{ with } n \geq 0. \quad \text{(4 pts)}$$

Answer first examination Math 132 (431).

Exercice 1: (12pts)

1). (3pts)

p	q	r	$\neg q$	$p\vee\neg q$	$\neg p$	$r\wedge\neg p$	$(p\vee\neg q)\to(r\wedge\neg p)$
T	T	T	F	T	F	F	F
T	T	F	F	T	F	F	F
T	F	T	T	T	F	F	F
T	F	F	T	T	F	F	F
F	T	T	F	F	T	T	T
F	T	F	F	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	F	F

Contingency.

2) (3pts)

$[(p\vee q)\wedge\neg p]\to q$

$\equiv ((p\wedge\neg p)\vee(q\wedge\neg p))\to q$

$\equiv [F\vee(q\wedge\neg p)]\to q$

$\equiv \neg(q\wedge\neg p)\vee q$

$\equiv (\neg q\vee p)\vee q \equiv (\neg q\vee q)\vee p$

$\equiv T\vee p \equiv T.$

3) (2pts)

p	q	$p\to q$	$p\wedge(p\to q)$	$\neg q$	$p\to\neg q$	Result
T	T	T	T	F	F	F
T	F	F	F	T	T	F
F	T	T	F	F	T	F
F	F	T	F	T	T	F

is a contradiction.

4) (2pts)

$[(\neg p\to q)\vee(\neg q\to r)]$

$\equiv [(p\vee q)\vee(q\vee r)]$

$\equiv p\vee q\vee r$

5) (2pts).

a). false; $x=\frac{1}{2}$; $\left(\frac{1}{2}\right)^2 \not< \left(\frac{1}{2}\right)^3$

b). True; $x=1$; $(1)^3=-1^3=-1$.

Exercice 2: (13pts)

1). p "$n+3$ is odd"

(3pts) q "$3n+4$ is even"

$p \Longleftrightarrow q$.

① $p\to q$: if $n+3$ is odd $\Rightarrow$ $3n+4$ is even.

$n+3$ is odd $\Rightarrow n+3=2k+1$; $k\in\mathbb{Z}$.

$\Rightarrow n=2k-2$,

$3n+4=3(2k-2)+4$

$=6k-6+4$

$=6k-2$

$=2(3k-1)=2t$; $t=3k-1\in\mathbb{Z}$.

$\Rightarrow 3n+4$ is even.

② $q\to p$; if $3n+4$ is even then $n+3$ is odd

by contraposition we prove that:

if $n+3$ is even; then $3n+4$ is odd.

$n+3$ is even $\Rightarrow n+3=2k$; $k\in\mathbb{Z}$.

$\Rightarrow n=2k-3$;

$\Rightarrow 3n+4=3(2k-3)+4$

$=6k-9+4$

$=6k-5$

$=6k-6+1$

$=2(3k-3)+1$

is odd

from ① and ② we have the result.

2) (2pts) we have $2x^3+3y^4+5z^2=144$.

We assume that $x>2$ and $y>2$ and $z>4$

$$\left.\begin{aligned} x>2 &\Rightarrow 2x^3>16 \\ y>2 &\Rightarrow 3y^4>48 \\ z>4 &\Rightarrow 5z^2>80 \end{aligned}\right\} 2x^3+3y^4+5z^2>144$$

that contradict the fact $2x^3+3y^4+5z^2=144$.

3) (4pts)

3). $P(n): 2+4+6+\dots+2n = n(n+1)$ $\forall n \geq 1$.

B.S: $P(1)$: L.H.S: 2.

R.H.S $= 1(1+1) = 2 =$ L.H.S.

So $P(1)$ is true.

I.S: Let $k \geq 1$, we prove that $P(k) \to P(k+1)$

we assume that $P(k)$ is true;

$2+4+6+\dots+2k = k(k+1)$ I.H.

and we prove that $P(k+1)$ is true.

$2+4+6+\dots+(2k+2) = (k+1)(k+2)$?

$2+4+6+\dots+2k+(2k+2)$

$= k(k+1) + 2k+2$

$= k(k+1) + 2(k+1) = (k+1)(k+2)$

so $P(k+1)$ is true

$\Rightarrow \forall n \geq 1$; $P(n)$ is true.

4) (4pts) $P(n): u_n = 3^n - 1$; $\forall n \geq 0$.

B.S: $P(0)$: $u_0 = 3^0 - 1 = 0$ True.

$P(1)$: $u_1 = 3^1 - 1 = 2$ True.

I.S: Let $k \geq 1$; we prove that

$(P(0) \wedge P(1) \wedge \dots \wedge P(k)) \to P(k+1)$

we assume that $P(0), P(1), \dots, P(k)$

are true and we prove that $P(k+1)$ is true

we prove that $u_{k+1} = 3^{k+1} - 1$.

$u_{k+1} = 4u_k - 3u_{k-1}$.

we have $P(k)$ true $\Rightarrow u_k = 3^k - 1$.

and $P(k-1)$ true $\Rightarrow u_{k-1} = 3^{k-1} - 1$.

so $u_{k+1} = 4u_k - 3u_{k-1}$

$= 4(3^k - 1) - 3(3^{k-1} - 1)$

$= 4\times 3^k - 3^k - 1$.

$= 3^k(4-1) - 1$

$= 3^k \times 3 - 1$

$u_{k+1} = 3^{k+1} - 1$

$\Rightarrow P(k+1)$ is true

$\Rightarrow \forall n \geq 0$: $u_n = 3^n - 1$.