King Saud University	College of Sciences	Department of Mathematics	
Second Examination	Math 132	Semester I (1443)	Time:1H30

Exercise 1: (9 pts)

1. Consider the set $X := \{x, z, \{x, \{x\}\}, \{y\}, \{z, \{\emptyset\}\}, \{y, x\}, \{z\}, \emptyset, \{\emptyset\}\}$.
   Determine whether each of the following six statements is true or false.
   (Justify your answer).
   (a) S_1: "$\{x, \{x\}\} \in X$". (1 pts)
   (b) S_2: "$\{x, \{x\}\} \subseteq X$".(1 pts)
   (c) S_3: "$\{y, \emptyset\} \subseteq X$". (1 pts)
   (d) S_4: "$\{\{\emptyset\}, \emptyset\} \subseteq X$". (1 pts)
   (e) S_5: "$\{z, \emptyset\} \in X$". (1 pts)
   (f) S_6: "$\{x, y, z, \emptyset\} \cap \{x, y, \{z\}, \{\emptyset\}\} \in X$". (1 pts)
2. Consider the following three sets $Y := \{1, 2, 3, 4\}$, $Z := \{2, 3\}$, and
   $T := \{(1, 2), (1, 4), (2, 2), (2, 4), (4, 4), (2, 3)\}$. Find the following sets:
   (a) $(Y \cap Z) \times Z$. (1 pts)
   (b) $T - (Y \times Z)$. (1 pts)
3. Let A and B be two sets such that $A - B = \{0, 1, 3, 5\}$, $B - A = \{2, 4, 6, 8\}$ and $A \cap B = \emptyset$. Find A and B. (1 pts)

Exercise 2: (16 pts)

Consider the set $A := \{0, 1, 2, 3, 4\}$.
Let R be the relation on the set A, such that,
$R := \{(0, 0), (0, 3), (0, 4), (1, 1), (1, 2), (2, 1), (2, 2), (3, 0), (3, 4), (4, 0), (4, 4)\}$.

1. Represent the relation R with a matrix. (1 pts)
2. Draw the digraph of the relation R. (1 pts)
3. Find $R^{-1} \circ R$ and $R \circ R^{-1}$. (3 pts)
4. Decide whether the relation R is reflexive, symmetric, anti-symmetric, or transitive. (Justify your answer) (4 pts)
5. Let S be the relation on the set A such that $S = R \cup \{(3, 3), (4, 3)\}$.
   (a) Prove that S is an equivalence relation on A. (3 pts)
   (b) Find the equivalence classes of S. (1 pts)
6. Let E be the relation on the set $\mathbb{Z}$ defined by:
   $a, b \in \mathbb{Z}$, $a \, E \, b \iff a - b$ is even.
   Prove that S is an equivalence relation on $\mathbb{Z}$. (3 pts)

Answer midterm 2
132 (431)

Exercice 1:

1) a) S_1 True because $\{x,\{x\}\}$ is an element of X. 1

b) S_2 false because $\{x\} \notin X$. 1

c) S_3 false because $y \notin X$. 1

d) S_4 True because $\phi \in X$, $\{\phi\} \in X$. 1

e) S_5 false $\{3,\phi\}$ not element of X. 1

f) S_6 True because 1

$\{x,y,3,\phi\} \cap \{x,y,\{3\},\{\phi\}\} = \{x,y\} \in X$.

2)

a) $Y \cap Z = \{2,3\}$.

$(Y \cap Z) \times Z = \{(2,2),(2,3),(3,2),(3,3)\}$ 1

b) $Y \times Z = \{(1,2),(1,3),(2,2),(2,3),(3,2),(3,2),(4,2),(4,3)\}$.

$T - (Y \times Z) = \{(1,4),(2,4),(4,4)\}$ 1

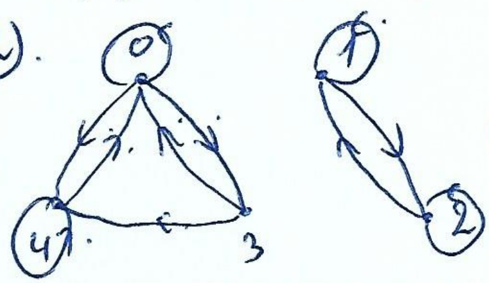
3) $A = \{0,1,3,5\}$; $B = \{2,4,6,8\}$. 1

Exercice 2:

1)

$$M_R = \begin{array}{c|ccccc} & 0 & 1 & 2 & 3 & 4 \\ \hline 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 & 1 \\ 4 & 1 & 0 & 0 & 0 & 1 \end{array}$$ 1

2).



1

3)

$R^{-1} = \{(0,0),(3,0),(4,0),(1,1),(2,1),(1,2),(2,2),(0,3),(4,3),(0,4),(4,4)\}$.

$R^{-1} \circ R = \{(0,0),(0,3),(0,4),(1,1),(1,2),(2,1),(2,2),(3,0),(3,3),(3,4),(4,0),(4,3),(4,4)\}$. 1,5

$R \circ R^{-1} = \{(0,0),(0,3),(0,4),(1,1),(1,2),(2,1),(2,2),(3,0),(3,3),(3,4),(4,0),(4,3),(4,4)\}$ 1,5

4)

* not reflexive $(0,0) \notin R$ 1
* not symmetric $(3,4) \in R$; $(4,3) \notin R$. 1
* not antisymmetric $(1,2) \in R$; $(2,1) \in R$. 1
* not transitive $(3,0) \in R$; $(0,3) \in R$. $(3,3) \notin R$ 1

5).

a) $(0,0) \in S$; $(1,1) \in S$; $(2,2) \in S$; $(3,3) \in S$; $(4,4) \in S$

$\Rightarrow S$ is reflexive. 1 1

* $(0,3) \in S$, $(3,0) \in S$.
  $(0,4) \in S$; $(4,0) \in S$.
  $(1,2) \in S$; $(2,1) \in S$.
  $(3,4) \in S$; $(4,3) \in S$.
  $\Rightarrow S$ is symmetric. 2

* $(0,3) \in S, (3,0) \in S \Rightarrow (0,0) \in S$. 1
  $(0,4) \in S, (4,0) \in S \Rightarrow (0,0) \in S$.
  $(0,3) \in S, (3,4) \in S \Rightarrow (0,4) \in S$.
  $(0,4) \in S; (4,3) \in S \Rightarrow (0,3) \in S$.
  $(1,2) \in S; (2,1) \in S \Rightarrow (1,1) \in S$.
  $(2,1) \in S; (1,2) \in S \Rightarrow (2,2) \in S$.
  $(3,0),(0,3) \in S \Rightarrow (3,3) \in S$.
  $(3,0) \in S; (0,4) \in S \Rightarrow (3,4) \in S$.
  $(3,4),(4,0) \Rightarrow (3,0)$
  $(3,4),(4,3) \Rightarrow (3,3)$.
  $(4,0),(0,3) \Rightarrow (4,3)$. 2
  $(4,0)(0,4) \Rightarrow (4,4)$
  $(4,3)(3,0) \Rightarrow (4,0)$ S transitive 3

①, ② and ③ $\Rightarrow$ S is an equivalence relation on A.

b) · the classes of S are $\{0,3,4\}$; $\{1,2\}$.

1

6) * $a \in \mathbb{Z}$; $a-a=0$ is even

$\Rightarrow aEa$

$\Rightarrow$ E is ~~an~~ reflexive ①.

1

* $a,b \in \mathbb{Z}$; $aEb \Rightarrow a-b$ even

$\Rightarrow b-a$ even

$\Rightarrow bEa$

$\Rightarrow$ E is symmetric ②.

1

* $a,b,c \in \mathbb{Z}$; aEb and bEc

$\Rightarrow a-b$ even and $b-c$ even

$\Rightarrow a-\cancel{b}+\cancel{b}-c$ is even

$\Rightarrow a-c$ is even

$\Rightarrow aEc$

$\Rightarrow$ E is transitive ③

1

$\Rightarrow$ ①, ② and ③ $\Rightarrow$ E is an equivalence relation on $\mathbb{Z}$.